BANGLADESH UNIVERSITY OF ENGINEERING & TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

Course No.: ME 346 (Mechanics of Machinery Sessional)

Problem Class: 01 (C1) **Date: 11 March, 2015**

Class Problems:

1. A spring-mass system has a natural frequency of 10 Hz. When the spring constant is reduced by 800 N/m, the frequency is altered by 45 percent. Find the mass and spring constant of the original system.

2. A helicopter landing gear consists of a metal framework rather than the coil spring based suspension system used in a fixed-wing aircraft. The vibration of the frame in the vertical direction can be modeled by a spring made of a slender bar, where the helicopter is modeled as ground. Here l = 0.4 m, E = 20×10^{10} N/m^2, and m = 100 kg. Calculate the cross-sectional area that should be used if the natural frequency is to be f_n = 500 Hz.

3. A shaft of length 0.75 m, supported freely at the ends, is carrying a body of mass 90 kg at 0.25m from one end. Find the natural frequency of transverse vibration. Assume E = 200 GN/m^2 and shaft diameter = 50mm.

4. A four-wheeled trolley car of total mass 2000 kg running on rails of 1.6 m gauge, rounds a curve of 30 m radius at 54 km/h. The track is banked at 8°. The wheels have an external diameter of 0.7 m and each pair with axle has a mass of 200 kg. The radius of gyration for each pair is 0.3 m. The height of centre of gravity of the car above the wheel base is 1 m. Determine, allowing for centrifugal force and gyroscopic couple actions, the pressure on each rail.

Solution

Problem # 1

A spring-mass system has a natural frequency of 10 Hz. When the spring constant is reduced by 800 N/m, the frequency is altered by 45 percent. Find the mass and spring constant of the original system.

Given,

$$\omega_n = 10\ Hz = 62.832\frac{rad}{s}$$

$$(\omega_n)_{new} = 0.55\ \omega_n = 34.5576\frac{rad}{s}$$

Now,

$$\omega_n = \sqrt{\frac{k}{m}}$$

Thus,

$$\frac{(\omega_n)_{new}}{\omega_n} = 0.55$$

$$\Rightarrow \sqrt{\frac{k-800}{k}} = 0.55$$

$$\Rightarrow k = 1146.95\frac{N}{m}$$

$$\therefore\ \sqrt{m} = \frac{\sqrt{1146.95}}{62.832}$$

$$\Rightarrow m = 0.2905\ kg$$

Problem # 2

A helicopter landing gear consists of a metal framework rather than the coil spring based suspension system used in a fixed-wing aircraft. The vibration of the frame in the vertical direction can be modeled by a spring made of a slender bar, where the helicopter is modeled as ground. Here l = 0.4 m, $E = 20 \times 10^{10}$ N/m^2, and m = 100 kg. Calculate the cross-sectional area that should be used if the natural frequency is to be f_n = 500 Hz.

Given,

$$\omega_n = 500\ Hz = 3142\frac{rad}{s}$$

Now,

$$\omega_n = \sqrt{\frac{k}{m}}$$

Considering a slender bar, the stiffness can be calculated as,

$$F = k\delta$$

$$\Longrightarrow k = \frac{F}{\delta} = \frac{F/_{A} \times A}{\epsilon l}$$

$$\Longrightarrow k = \frac{\sigma A}{\epsilon l} = \frac{EA}{l}$$

Thus,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{EA}{lm}}$$

$$\Longrightarrow 3142 = \sqrt{\frac{20 \times 10^{10} \times A}{0.4 \times 100}}$$

$$\Longrightarrow A = 0.0019\ m^2 = 19\ cm^2$$

Problem # 3

A shaft of length 0.75 m, supported freely at the ends, is carrying a body of mass 90 kg at 0.25m from one end. Find the natural frequency of transverse vibration. Assume E = 200 GN/m^2 and shaft diameter = 50mm.

Given,

l = 0.75 m

m = 90 kg

a = 0.25 m

E = 200 GN/m^2

d = 50 mm = 0.05 m

Now,

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times (0.05)^4\ m^4$$

$$\Longrightarrow I = 0.307 \times 10^{-6}\ m^4$$

And,

$$\delta = \frac{Wa^2b^2}{3EIl} = \frac{90 \times 9.81 \times (0.25)^2 \times (0.5)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 0.75}$$

$$\Longrightarrow \delta = 0.1 \times 10^{-3}\ m$$

Thus,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{0.4985}{\sqrt{0.1 \times 10^{-3}}} = 49.85\ Hz$$

Problem # 4

A four-wheeled trolley car of total mass 2000 kg running on rails of 1.6 m gauge, rounds a curve of 30 m radius at 54 km/h. The track is banked at 8°. The wheels have an external diameter of 0.7 m and each pair with axle has a mass of 200 kg. The radius of gyration for each pair is 0.3 m. The height of centre of gravity of the car above the wheel base is 1 m. Determine, allowing for centrifugal force and gyroscopic couple actions, the pressure on each rail.

Given,

m_v = 2000 kg

x = 1.6 m

v = 54 km/h = 15 m/s

θ = 8°

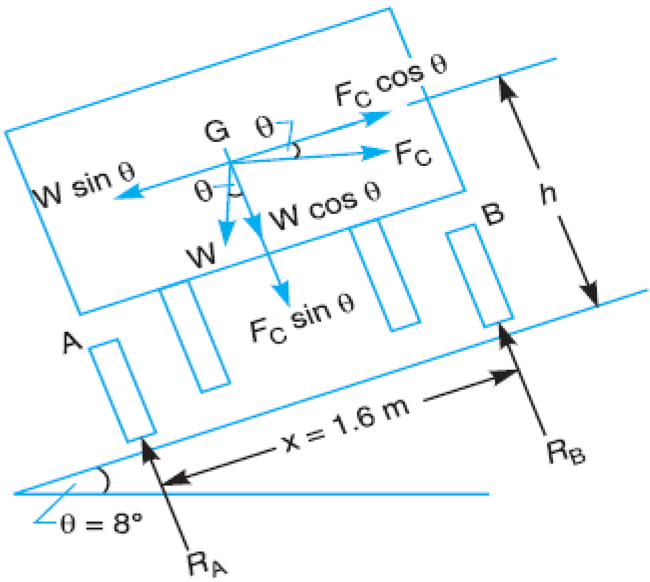
d_w = 0.7 m

m_a = 200 kg

h = 1m

k = 0.3 m

R = 30 m



Now,

$$\sum F = R_A + R_B = W\,cos\theta + F_c\,sin\theta = mg\,cos\theta + \frac{m_v v^2}{R} sin\theta$$

$$\Rightarrow R_A + R_B = 2000 \times 9.81\cos 8° + \frac{2000 \times (15)^2}{30}\sin 8°$$

$$\Rightarrow R_A + R_B = 21{,}518\ N$$

Now, taking moments about B,

$$R_A \times x = (W\,cos\theta + F_c\,sin\theta)\frac{x}{2} + W\,sin\theta \times h - F_c\,cos\theta \times h$$

$$\Rightarrow R_A = \left[2000 \times 9.81\cos 8° + \frac{2000 \times (15)^2}{30}\sin 8°\right] \times \frac{1}{2} + 2000 \times 9.81\sin 8° \times 1 - \frac{2000 \times (15)^2}{30}\cos 8° \times \frac{1}{1.6}$$

$$\Rightarrow R_A = 3{,}182\ N$$

$$\therefore R_B = 18{,}226\ N$$

Now, angular speed of wheel,

$$\omega_w = \frac{v}{r_w} = \frac{15}{0.35} = 42.86\frac{rad}{s}$$

And, angular speed of precession,

$$\omega_p = \frac{v}{R} = \frac{15}{30} = 0.5\frac{rad}{s}$$

Thus, gyroscopic couple,

$$C = I\ \omega_w \cos\theta\ \times\ \omega_p = m\,k^2\ \omega_w \cos\theta\ \omega_p$$

$$\Longrightarrow C = 200\ \times\ (0.3)^2\ \times 42.86 \cos 8°\ \times 0.5 = 382\ N - m$$

Pressure due to the gyroscopic couple,

$$P = \frac{C}{x} = \frac{382}{1.6} = 238.75\ N$$

Pressure on the inner rail,

$$P_I\ =\ R_A - P = 2{,}943.25\ N$$

Pressure on the outer rail,

$$P_O\ =\ R_B + P = 18{,}574.75\ N$$
