BANGLADESH UNIVERSITY OF ENGINEERING & TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

Course No.: ME 346 (Mechanics of Machinery Sessional)

Problem Class: 03 (Torsional Vibration and Damped Vibration) **Date: 5 April, 2015**

Class Problems:

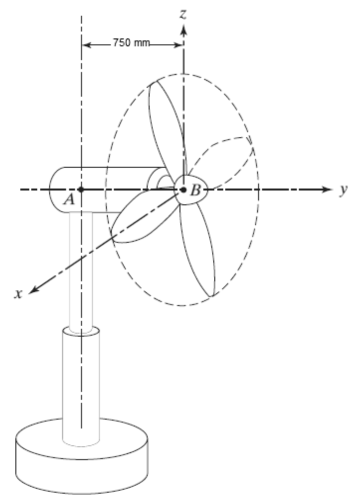
1. Three motors A, B and C having moment of inertia of 2000; 6000 and 3500 kg-m^2 respectively are carried on a uniform shaft of 0.35 m diameter. The length of the shaft between the rotors A and B is 6 m and between B and C is 32 m. Find the natural frequency of the torsional vibrations. The modulus of rigidity for the shaft material is 80 GN/m^2.

2. An electric motor rotating at 1500 rpm drives a centrifugal pump at 500 rpm through a single stage reduction gearing. The moments of inertia of the electric motor and the pump impeller are 400 kg-m^2 and 1400 kg-m^2 respectively. The motor shaft is 45 mm in diameter and 180 mm long. The pump shaft is 90 mm in diameter and 450 mm long.

   Determine the frequency of torsional oscillations of the system, neglecting the inertia of the gears. The modulus of rigidity for the shaft material is 84 GN/m^2.

3. One of the blades of an electric fan is removed (as shown by dotted lines in Figure). The steel shaft AB, on which the blades are mounted, is equivalent to a uniform shaft of diameter 25 mm and length 750 mm. Each blade can be modeled as a uniform slender rod of weight 4 kg and length 300 mm. Determine the natural frequency of vibration of the remaining three blades about the y-axis.

   [Modulus of rigidity for steel material is 80 GN/m^2.]

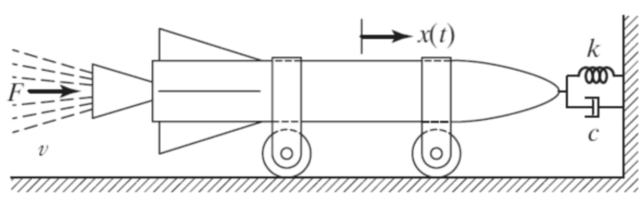


4. In a static firing test, a rocket is anchored to a rigid wall by a spring-damper system, as shown in figure. The thrust acting on the rocket reaches its maximum value F in a negligibly short time and remains constant until the burnout time t_0. The thrust acting on the rocket is given by $F = m_0 v$, where m_0 is the constant rate at which fuel is burnt and v is the velocity of the jet stream. The initial mass of the rocket is M, so that mass at any time t is given by $m = M - m_0 t$, $0 \le t \le t_0$.

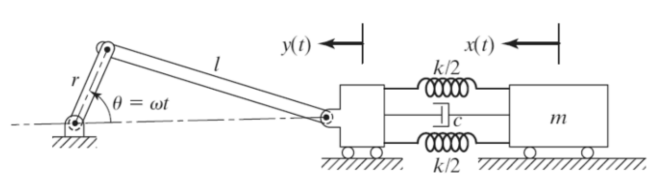
   If the data are $k = 7.5 \times 10^6$ N/m, $c = 0.1 \times 10^6$ N-s/m, $m_0 = 10$ kg/s, $v = 2000$ m/s, $M = 2000$ kg, and $t_0 = 100$s

   (1) derive the equation of motion of the rocket and

   (2) find the maximum steady-state displacement of the rocket by assuming an average (constant) mass of $\left(M - \frac{1}{2} m_0 t_0\right)$.



5. A slider-crank mechanism is used to impart motion to the base of a spring-mass-damper system, as shown in figure. Approximateing the base motion y(t) as a series of harmonic functions, find the response of the mass for m = 1 kg, c = 10 N-s/m, k = 100 N/m, r = 10 cm, l = 1m and ω = 100 rad/s.



Solution

Due to constant term,

$$x(t) = \frac{F_0}{k}\left[1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \cos(\omega_d t - \varphi)\right]$$

where, $\varphi = \tan^{-1}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$

Due to sinusoidal term, $(F_0 \sin \Omega t)$

$$x(t) = X \sin(\Omega t - \varphi_0)$$

where, $X = \frac{F_0}{\sqrt{(k - m\Omega^2)^2 + c^2\Omega^2}}$

$$\varphi_0 = \tan^{-1}\left(\frac{c\Omega}{k - m\Omega^2}\right)$$

$$\Omega = 2\omega$$

Due to cosine term, $(F_0 \cos \Omega t)$

$$x(t) = X \cos(\Omega t - \varphi_0)$$

For given data, $\xi = \frac{c}{2\sqrt{mk}} = \frac{10}{2\sqrt{1 \times 100}} = 0.5$

$\frac{r}{l} = 0.1$ $\quad \omega_d = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{1}} = 10$ rad/s

$\omega = 100$ rad/s

$\Omega = 200$ rad/s