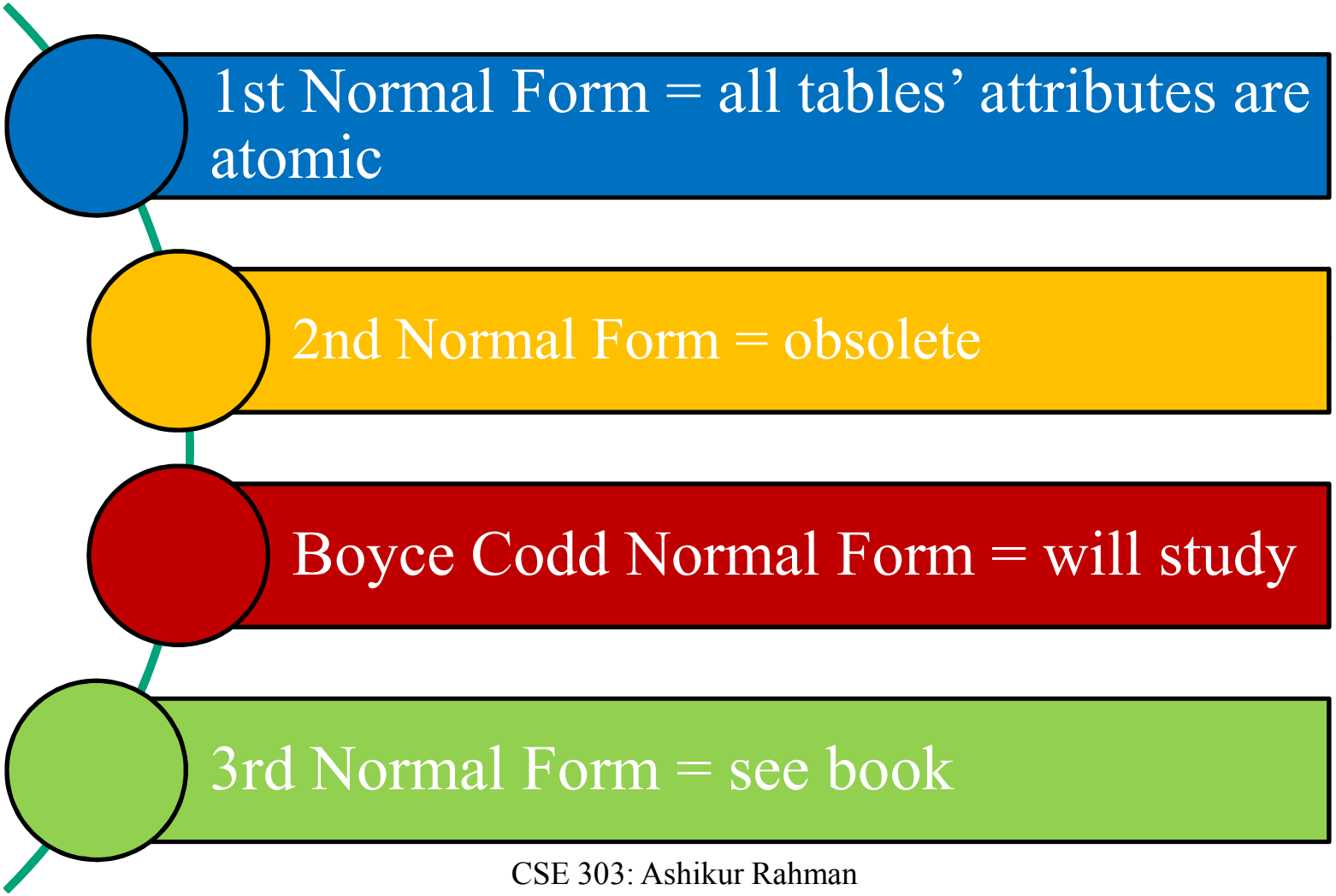
CSE 303: Database

Lecture 10

Chapter 3: Design Theory of Relational Database

Outline


1st Normal Form = all tables' attributes are atomic
2nd Normal Form = obsolete
Boyce Codd Normal Form = will study
3rd Normal Form = see book

First Normal Form (1NF)

- A database schema is in First Normal Form (1NF) if the **domain** of each attribute contains only **atomic** values, and the value of each attribute contains only a **single value** from that **domain.**

First Normal Form (1NF)

Customer

Customer ID	First Name	Surname	Telephone Number
123	Robert	Ingram	555-861-2025
456	Jane	Wright	555-403-1659
789	Maria	Fernandez	555-808-9633

Not in First Normal Form (1NF)

Customer

Customer ID	First Name	Surname	Telephone Number
123	Robert	Ingram	555-861-2025
456	Jane	Wright	555-403-1659 555-776-4100
789	Maria	Fernandez	555-808-9633

Now in First Normal Form (1NF)

Customer

Customer ID	First Name	Surname	Telephone Number
123	Robert	Ingram	555-861-2025
456	Jane	Wright	555-403-1659
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789	Maria	Fernandez	555-808-9633

Now in First Normal Form (1NF)

Customer ID	First Name	Surname
123	Robert	Ingram
456	Jane	Wright
789	Maria	Fernandez

Customer ID	Telephone Number
123	555-861-2025
456	555-403-1659
456	555-776-4100
789	555-808-9633

First Normal Form (1NF)

A database schema is in First Normal Form if all tables' attributes contain only **atomic** values.

Student

Name	GPA	Courses
Alice	3.8	Math DB OS
Bob	3.7	DB OS
Carol	3.9	Math OS

May need to add keys

Student

Name	GPA
Alice	3.8
Bob	3.7
Carol	3.9

Takes

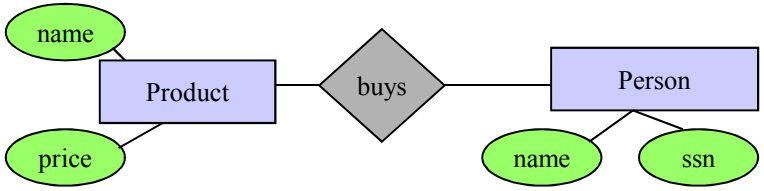
Student	Course
Alice	Math
Carol	Math
Alice	DB
Bob	DB
Alice	OS
Carol	OS

Course

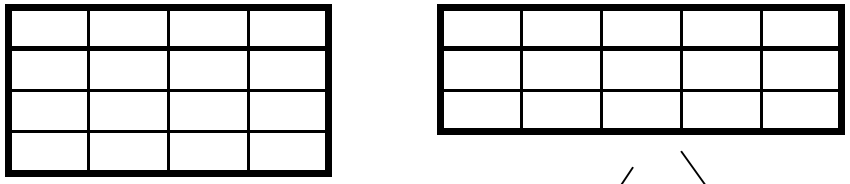
Course
Math
DB
OS

Relational Schema Design

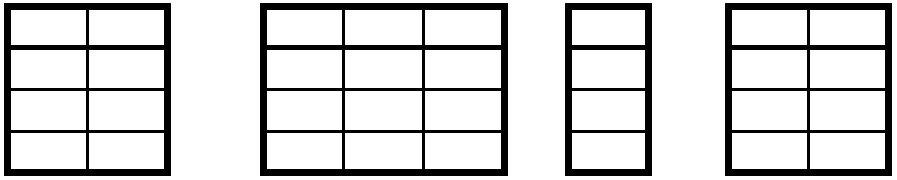
Conceptual Model:



Relational Model (in 1NF)
plus FD's



Normalization:
Eliminates ***anomalies***



Data Anomalies

When a database is poorly designed we get anomalies:

Redundancy: data is repeated

Update anomalies: need to change in several places

Delete anomalies: may lose data when we don't want

Relational Schema Design

Recall set attributes (persons with several phones):

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

One person may have multiple phones, but lives in only one city

Anomalies:

- Redundancy = repeated data
- Update anomalies = Fred moves to "Bellevue"
- Deletion anomalies = Joe deletes his phone number: what is his city ?

Relation Decomposition

Break the relation into two:

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

Name	SSN	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

SSN	PhoneNumber
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121

Anomalies are gone:

- No more repeated data
- Easy to move Fred to “Bellevue” (how?)
- Easy to delete all Joe’s phone numbers (how?)

Relational Schema Design (or Logical Design)

Main idea:

- Start with some relational schema
- Find out its ***functional dependencies***
- Use them to design a better relational schema

Functional Dependencies

- A form of constraint
  - hence, part of the schema
- Finding them is part of the database design
- Also used in normalizing the relations

Functional Dependencies

Definition:

If two tuples agree on the attributes

$A_1, A_2, \ldots, A_n$

then they must also agree on the attributes

$B_1, B_2, \ldots, B_m$

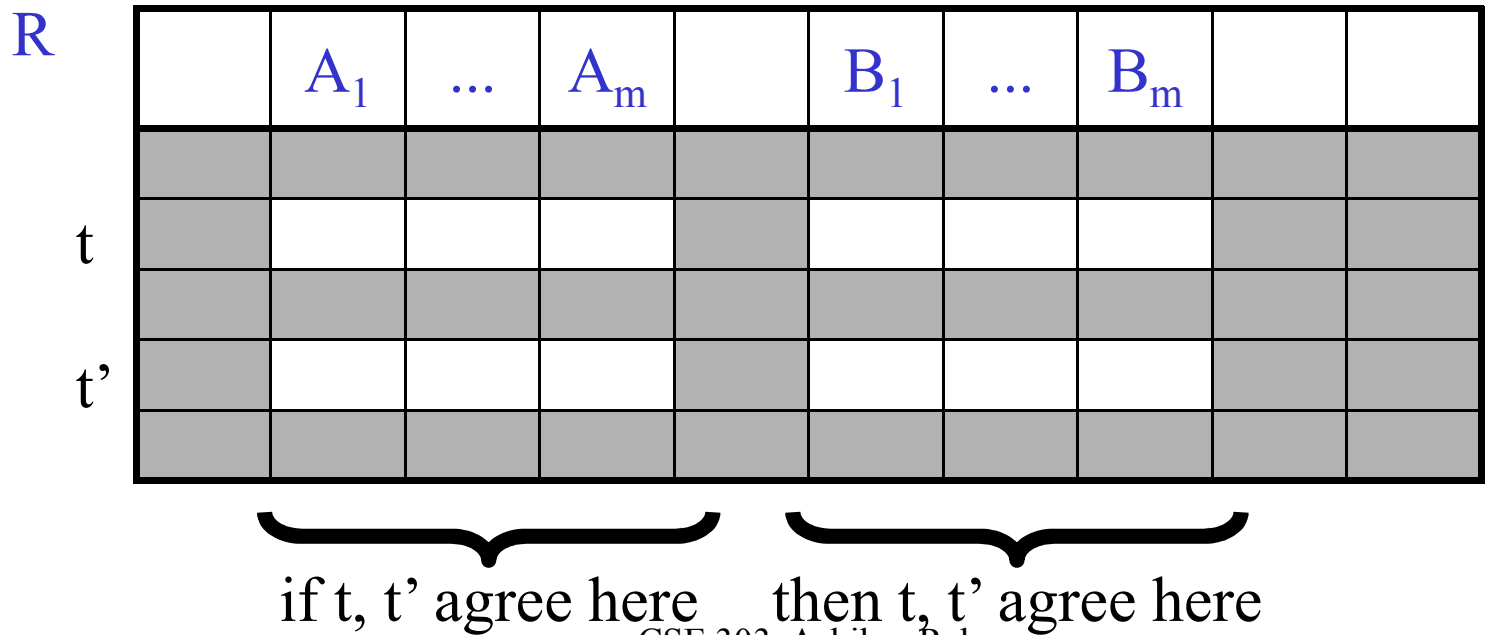
Formally:

$A_1, A_2, \ldots, A_n \rightarrow B_1, B_2, \ldots, B_m$

When Does an FD Hold

Definition: $A_1, ..., A_m \rightarrow B_1, ..., B_n$ holds in R if:

$\forall t, t' \in R, (t.A_1=t'.A_1 \wedge ... \wedge t.A_m=t'.A_m \Rightarrow t.B_1=t'.B_1 \wedge ... \wedge t.B_n=t'.B_n)$



Example: Movie table

Title	Year	Length	Genre	StudioName	StarName
Star Wars	1977	124	SciFi	Fox	Carrie Fisher
Star Wars	1977	124	SciFi	Fox	Mark Hamill
Star Wars	1977	124	SciFi	Fox	Harrison Ford
Gone With the Wind	1939	231	Drama	MGM	Vivien Leigh
Wayne's World	1992	95	Comedy	Paramount	Dana Carvey
Wayne's World	1992	95	Comedy	Paramount	Mike Meyers

title, year → length

title, year → genre

title, year → studioName

title, year → length, genre, studioName

Example: Movie table

Title	Year	Length	Genre	StudioName	StarName
Star Wars	1977	124	SciFi	Fox	Carrie Fisher
Star Wars	1977	124	SciFi	Fox	Mark Hamill
Star Wars	1977	124	SciFi	Fox	Harrison Ford
Gone With the Wind	1939	231	Drama	MGM	Vivien Leigh
Wayne's World	1992	95	Comedy	Paramount	Dana Carvey
Wayne's World	1992	95	Comedy	Paramount	Mike Meyers

How about?

title, year → starName

Examples

An FD holds, or does not hold on an instance:

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

EmpID → Name, Phone, Position

but not Name → EmpID

or Name → Phone

Example

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876 ←	Salesrep
E1111	Smith	9876 ←	Salesrep
E9999	Mary	1234	Lawyer

Position → Phone

Example

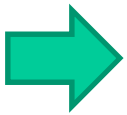
EmpID	Name	Phone	Position
E0045	Smith	1234 →	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234 →	Lawyer

but not Phone → Position

Inferring other dependencies from a set of FDs

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office-supp.	59

name → color
category → department
color, category → price



name, category → price

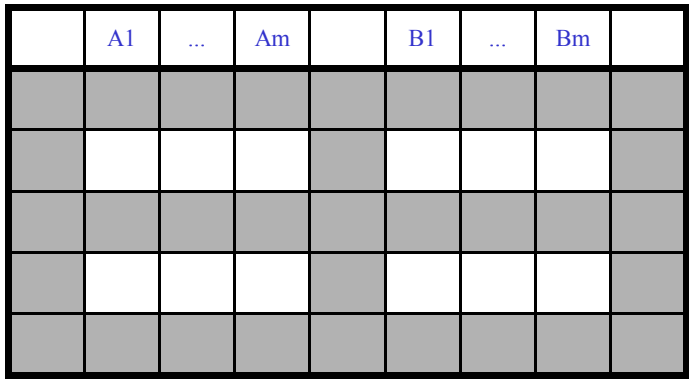
Armstrong's Rules (1/3)

$A_1, A_2, \ldots, A_n \rightarrow B_1, B_2, \ldots, B_m$

Is equivalent to

$A_1, A_2, \ldots, A_n \rightarrow B_1$
$A_1, A_2, \ldots, A_n \rightarrow B_2$
. . . . .
$A_1, A_2, \ldots, A_n \rightarrow B_m$

Splitting rule and Combing rule



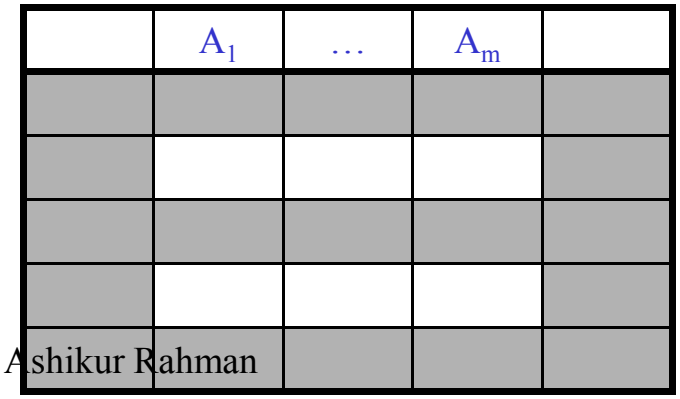
Armstrong's Rules (2/3)

$A_1, A_2, \ldots, A_n \rightarrow A_i$

Trivial Rule

where i = 1, 2, ..., n

Why ?



Armstrong's Rules (3/3)

Transitive Closure Rule

If $A_1, A_2, \ldots, A_n \rightarrow B_1, B_2, \ldots, B_m$

and $B_1, B_2, \ldots, B_m \rightarrow C_1, C_2, \ldots, C_p$

then $A_1, A_2, \ldots, A_n \rightarrow C_1, C_2, \ldots, C_p$

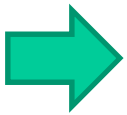
Why ?

	A_1	…	A_m		B_1	…	B_m		C_1	...	C_p	

Inferring other dependencies from a set of FDs

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office-supp.	59

name → color
category → department
color, category → price

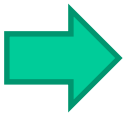


name, category → price

Example (continued)

Start from the following FDs:

1. name → color
2. category → department
3. color, category → price



name, category → price

THIS IS TOO HARD!
Let's see an easier way.

Infer the following FDs:

Inferred FD	Which Rule did we apply ?
4. name, category → name	Trivial
5. name, category → color	Transitive 1, 4
6. name, category → category	Trivial
7. name, category → color, category	Split/combine
8. name, category → price	Transitive 7, 3

Closure of a set of Attributes

Given a set of attributes $A_1, \ldots, A_n$ and a set of FDs

The **closure**, $\{A_1, \ldots, A_n\}^+$ = the set of attributes B s.t. $A_1, \ldots, A_n \rightarrow B$

Closures Example

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office-supp.	59

Example:

name → color
category → department
color, category → price

Closures:

$name^+ = \{name, color\}$

$\{name, category\}^+ = \{name, category, color, department, price\}$

$color^+ = \{color\}$

Closure Algorithm

X={A1, …, An}.

Repeat until X doesn't change **do**:

if $B_1, \ldots, B_n \rightarrow C$ is a FD **and**
$B_1, \ldots, B_n$ are all in X
then add C to X.

Example:

name → color
category → department
color, category → price

$\{name, category\}^+ =$
{ name, category, color, department, price }

Hence: name, category → color, department, price

More Examples

In class:

R(A,B,C,D,E,F)

A, B	→	C
B, C	→	A, D
D	→	E
C, F	→	B

Compute $\{A,B\}^+$ X = {A, B, }

Compute $\{A, F\}^+$ X = {A, F, }

More Examples

In class:

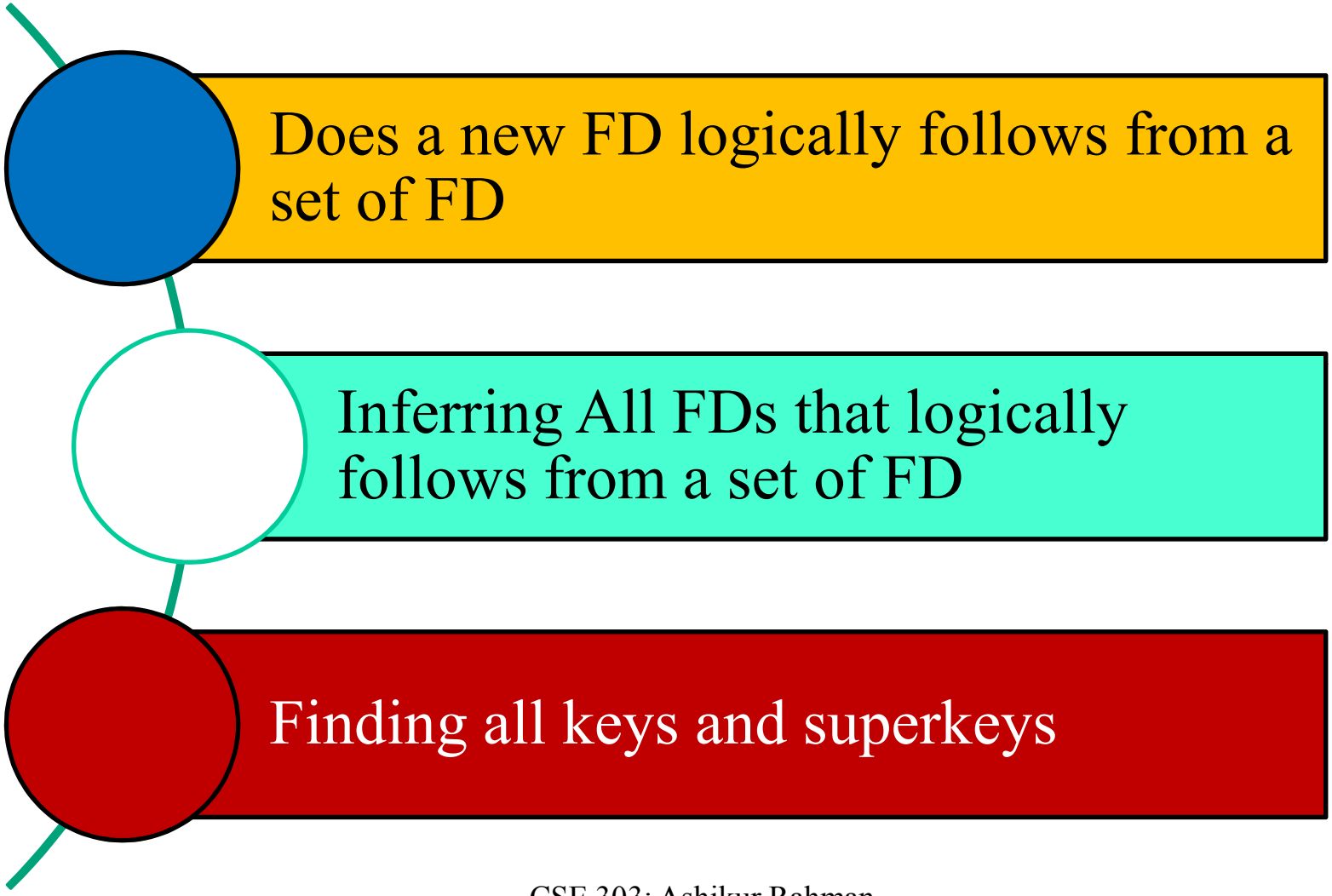
R(A,B,C,D,E,F)

A, B	→	C
B, C	→	A, D
D	→	E
C, F	→	B

Compute $\{A,B\}^+$ $X = \{A, B, C, D, E\}$

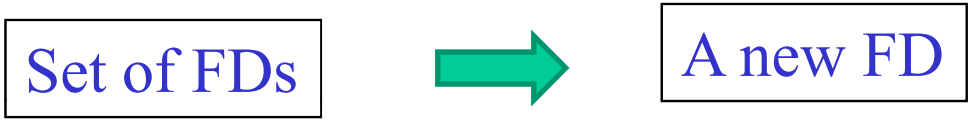
Compute $\{A, F\}^+$ $X = \{A, F\}$

Application of Closures



(1) Does a new FD logically follows from a set of FDs?

$R(A_1, A_2, ..., A_m)$

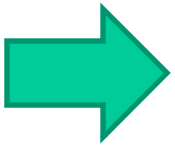


- To check if $X \rightarrow A$ (new FD)
  - Using given set of FDs Compute X^+ i.e., {left side}$^+$
  - Check if $A \in X^+$

Example

R(A,B,C,D)

A, B	→	C
B, C	→	D
C, D	→	A
A, D	→	B



A, D → C

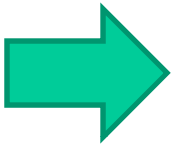
1. Compute $\{A,D\}^+$

2. If it contains C then the new FD logically follows

Example

R(A,B,C,D)

A, B → C
B, C → D
C, D → A
A, D → B



B, D → A

1. Compute $\{B, D\}^+ = \{B, D\}$

2. A is not a member of the set, hence it doesn't logically follow

Using Closure to Infer ALL FDs

Example:

A, B	→	C
A, D	→	B
B	→	D

Step 1: Compute X^+, for every $X \subseteq \{A,B,C,D\}$:

$A+ = A$, $B+ = BD$, $C+ = C$, $D+ = D$

$AB+ = ABCD$, $AC+ = AC$, $AD+ = ABCD$,

$BC+ = BCD$, $BD+ = BD$, $CD+ = CD$

$ABC+ = ABD+ = ACD^+ = ABCD$ (no need to compute– why ?)

$BCD^+ = BCD$, $ABCD+ = ABCD$

Step 2: Enumerate all FD's $X \rightarrow Y$, s.t. $Y \subseteq X^+$ and $X \cap Y = \emptyset$:

AB → CD, AD→BC, BC →D, ABC → D, ABD → C, ACD → B

Another Example

- Enrollment(student, major, course, room, time)

  student → major

  major, course → room

  course → time

  What else can we infer ? [in class, or at home]

Application of Closure: Finding Keys

- A **superkey** is a set of attributes A_1, ..., A_n s.t. for any other attribute B, we have A_1, ..., $A_n \rightarrow B$

- A **key** is a minimal superkey
  - i.e. set of attributes which is a superkey and for which no subset is a superkey

How many Superkeys?

Suppose R is a relation with attributes A_1, A_2,….A_n.
As a function of n, tell how many superkeys R has, if:

a) The only key is A_1

b) The only keys are A_1 and A_2

How many Superkeys?

Suppose R is a relation with attributes A_1, A_2,….A_n.
As a function of n, tell how many superkeys R has, if:

a) The only key is A_1 → 2^{n-1}

b) The only keys are A_1 and A_2 → 3.2^{n-2}

Computing (Super)Keys

- Compute X^+ for all sets X
- If X^+ = all attributes, then X is a key
- List only the minimal X's

Example 1

R(A,B,C,D)

A, B	→	C
B, C	→	D
C, D	→	A
A, D	→	B

What are all the keys?

What are all the superkeys that are not keys?

Example

R(A,B,C,D)

A, B → C
B, C → D
C, D → A
A, D → B

Keys:
AB , AD, BC, CD

$A^+ = A$, $B^+ = B$, $C^+ = C$, $D^+ = D$
$AB^+ = ABCD$, $AC^+ = AC$, $AD^+ = ABCD$,
$BC^+ = ABCD$, $BD^+ = BD$, $CD^+ = ABCD$
$ABC^+ = ABD^+ = ACD^+ = ABCD$ (no need to compute– why ?)
$BCD^+ = ABCD$, $ABCD^+ = ABCD$

Example

R(A,B,C,D)

A, B	→	C
B, C	→	D
C, D	→	A
A, D	→	B

Superkeys that are not Keys:
ABC , ABD, BCD, ACD, ABCD

$A^+ = A$, $B^+ = B$, $C^+ = C$, $D^+ = D$

$AB^+ = ABCD$, $AC^+ = AC$, $AD^+ = ABCD$,

$BC^+ = ABCD$, $BD^+ = BD$, $CD^+ = ABCD$

$ABC^+ = ABD^+ = ACD^+ = ABCD$ (no need to compute– why ?)

$BCD^+ = ABCD$, $ABCD^+ = ABCD$

Example 2

A, B → C
A, D → B
B → D

Keys:
AB , AD

A+ = A, B+ = BD, C+ = C, D+ = D
AB+ =ABCD, AC+=AC, AD+=ABCD,
BC+=BCD, BD+=BD, CD+=CD
ABC+ = ABD+ = ACD^+ = ABCD (no need to compute– why ?)
BCD^+ = BCD, ABCD+ = ABCD

Example 2

A, B	→	C
A, D	→	B
B	→	D

Superkeys that are not Keys:
ABC , ABD, ACD, ABCD

A+ = A, B+ = BD, C+ = C, D+ = D
AB+ =ABCD, AC+=AC, AD+=ABCD,
BC+=BCD, BD+=BD, CD+=CD
ABC+ = ABD+ = ACD^{+} = ABCD (no need to compute– why ?)
BCD^{+} = BCD, ABCD+ = ABCD

Example

Product(name, price, category, color)

name, category → price
category → color

What is the key ?

Example

Product(name, price, category, color)

name, category → price
category → color

What is the key ?

(name, category) + = name, category, price, color

Hence (name, category) is a key

Examples of Keys

Enrollment(student, address, course, room, time)

student → address
room, time → course
student, course → room, time

(find keys at home)

Key or Keys ?

Can we have more than one key ?

Given R(A,B,C) define FD's s.t. there are two keys

AB→C BC→A	or	A→BC B→AC

what are the keys here ?

Can you design FDs such that there are *three* keys ?

Eliminating Anomalies

Main idea:

- $X \rightarrow A$ is OK if X is a (super)key
- $X \rightarrow A$ is not OK otherwise

Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

SSN → Name, City

What is the key?

{SSN, PhoneNumber}

Hence SSN → Name, City is a "bad" dependency

Boyce-Codd Normal Form (BCNF)

A simple condition for removing anomalies from relations:

A relation R is in BCNF if:

If $A_1, ..., A_n \rightarrow B$ is a non-trivial dependency

in R, then $\{A_1, ..., A_n\}$ is a superkey for R

In other words: there are no "bad" FDs, that is the left side of every nontrivial FD must be a super key.

Equivalently:

$\forall$ X, either $(X^+ = X)$ or $(X^+ = \text{all attributes})$

All two-attribute relations are in BCNF

R(A,B)

Case 1: Suppose there is no dependency

Nothing is being violated, so fine

Case 2: Suppose A → B is the only dependency

$A^+ = AB$, so left side of A → B is a key

Case 3: Suppose B → A is the only dependency

$B^+ = AB$, so left side of B → A is a key

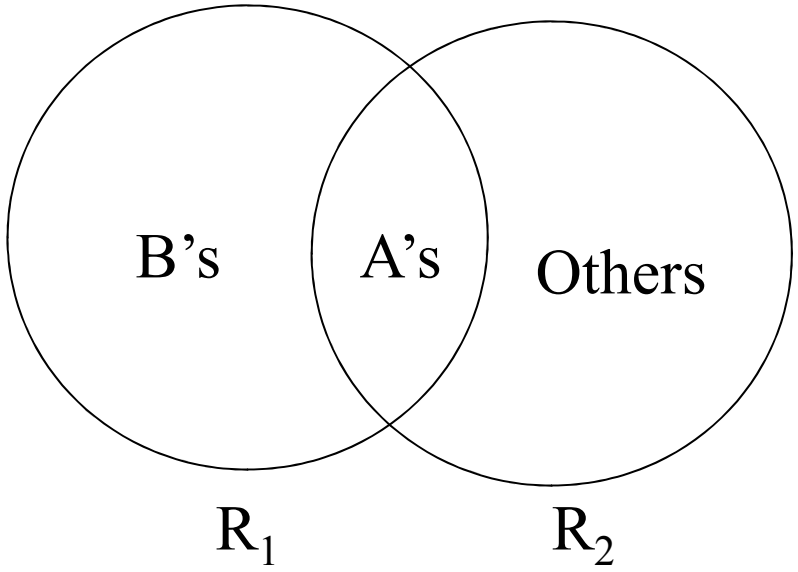
Case 4: Suppose A→ B and B → A are two dependencies

$A^+ = AB$, $B^+ = AB$,

so left side of B → A is a key and left side of A→B is also a key

BCNF Decomposition Algorithm

repeat
  choose $A_1, \ldots, A_m \rightarrow B_1, \ldots, B_n$ that violates BCNF
  split R into $R_1(A_1, \ldots, A_m, B_1, \ldots, B_n)$ and $R_2(A_1, \ldots, A_m, [\text{others}])$
  continue with both R_1 and R_2
until no more violations



In practice, we have
a better algorithm (coming up)

Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

SSN → Name, City

What is the key?

{SSN, PhoneNumber}

use SSN → Name, City
to split

R1(SSN, Name, City)
R2(SSN, PhoneNumber)

Example

Name	SSN	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

SSN → Name, City

SSN	PhoneNumber
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121
987-65-4321	908-555-1234

Let's check anomalies:

- Redundancy ?
- Update ?
- Delete ?

BCNF Decomposition Algorithm

BCNF_Decompose(R)

find X s.t.: $X \neq X^+ \neq$ [all attributes]

if (not found) **then** "R is in BCNF"

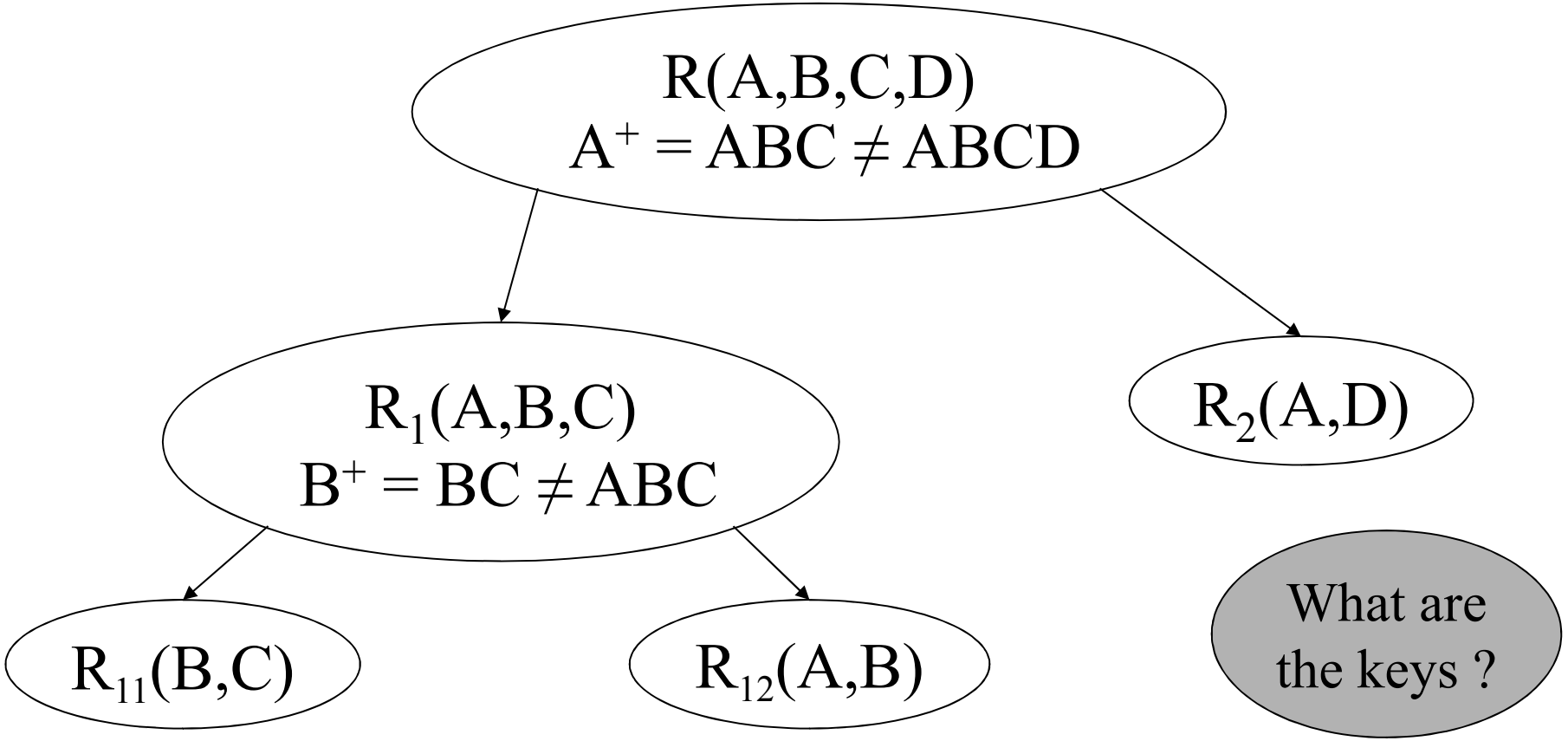
let $Y = X^+ - X$
let Z = [all attributes] - X^+
decompose R into R1($X \cup Y$) and R2($X \cup Z$)
continue to decompose recursively R1 and R2

R(A,B,C,D)

Find X s.t.: $X \neq X^+ \neq$ [all attributes]

$A \rightarrow B$
$B \rightarrow C$

Example



Find X s.t.: X ≠X^+ ≠ [all attributes]

Example2: BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN → name, age

age → hairColor

Iteration 1: Person

SSN^+ = SSN, name, age, hairColor

Decompose into: P(SSN, name, age, hairColor)

Phone(SSN, phoneNumber)

Iteration 2: P

SSN^+ = SSN, name, age, hairColor

age+ = age, hairColor

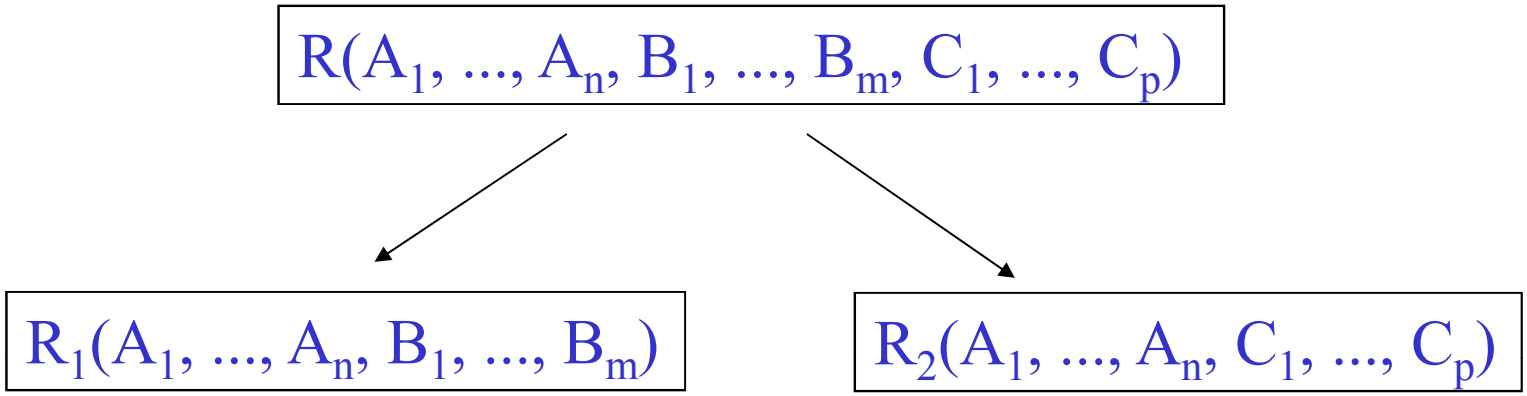
Decompose: People(SSN, name, age)

Hair(age, hairColor)

Phone(SSN, phoneNumber)

What are the keys ?

Decompositions in General



R_1 = projection of R on $A_1, ..., A_n, B_1, ..., B_m$
R_2 = projection of R on $A_1, ..., A_n, C_1, ..., C_p$