# Mathematical Analysis of Self-pruning and a New Dynamic Probabilistic Broadcast for MANETs

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***Abstract*—Self-pruning broadcasting algorithm exploits neighbor knowledge to reduce redundant retransmissions in mobile ad hoc wireless networks (MANETs). Although in self-pruning, only a subset of nodes forward the message based on certain forwarding rule, it belongs to one of the reliable broadcasting algorithm category where a broadcast message is guaranteed (at least algorithmically) to reach all the nodes in the network. In this paper, we develop an analytical model to determine expected number of forwarding nodes required to complete a broadcast in self-pruning algorithm. The derived expression is a function of various network parameters (such as, network density and distance between nodes) and radio transceiver parameters (such as transmission range). Moreover, the developed mathematical expression provides us a better understanding of the highly complex packet forwarding pattern of self-pruning algorithm and valuable insight to design a new broadcasting heuristic. The proposed new heuristic is a dynamic probabilistic broadcast where rebroadcast probability of each node is dynamically determined from a developed mathematical expression. Extensive simulation experiments have been conducted to validate the accuracy of the analytical model, as well as, to evaluate the efficiency of the proposed heuristic. Performance analysis shows that the proposed heuristic outperforms the static probabilistic broadcasting algorithm and an existing solution proposed by Bahadili.**

## I. Introduction

A mobile ad hoc wireless network (MANET) is a collection of low power wireless mobile hosts forming a temporary network without the aid of any global topological information or infrastructure. Broadcasts and unicasts are two unique ways for data dissemination in MANETs. Due to the dynamic nature of MANETs, broadcasting is more frequent compared to the wired network. Broadcasting is used for many fundamental operations in MANETs, such as, route discovery process in several routing protocols [1], [2] and [3] or sending an error message to erase invalid routes in a mobile environment. Some protocols even use broadcasting for actual data transmissions.

The most *naive* approach for broadcasting is flooding where a broadcast packet is forwarded exactly once by every node [4] in the network. Although theoretically, such blind flooding ensures a complete coverage, in reality, such naive flooding may cause serious redundancy, contention, and collision, which is known as broadcast storm problem [5] in the literature. Moreover, in wireless environment each node operates in promiscuous receive mode which often makes such blind flooding undesirable. Therefore, a variety of broadcasting techniques have been proposed to reduce redundant packet forwarding generated by flooding [6], [7] and [8]. All those techniques can be broadly classified into two categories:- (i) *unreliable* broadcasts, and (ii) *reliable* broadcasts. In an unreliable broadcast, a node may miss a broadcast packet because the protocol (intelligently) reduces redundancy by inhibiting packet forwarding from the nodes which find themselves "less-effective" as intermediate forwarder. The *effectiveness* of a node is often measured based on some preset probability values or counter values, or based on distance and location [5]. On the contrary, the reliable protocols are based on the so-called concept of "connected dominating sets" (CDS) and ensures that every node in the network receives a broadcast. Many reliable broadcasting algorithms have been proposed over the past decade [8], [9], [10] and [11]. One of the widely used reliable broadcast algorithms known as self-pruning uses neighborhood knowledge [9] for forwarding packets and significantly reduces redundant data transmissions. So far, the evaluation of self-pruning is mainly based on experiments and lacks detailed theoretical analysis. One inherent reason lies on the fact that even a simple broadcasting algorithm like self-pruning typically generates highly complex packet forwarding patterns and the existing protocols were only targeting simple heuristics. In this paper, we mathematically analyze self-pruning algorithm and quantify the forwarding probability of a node located at a certain distance from the source of a broadcast. Using the forwarding probability, we build mathematical model that estimates average fraction of neighbors forwarding the packet within a nodes neighborhood. The developed mathematical model gives us insight to the efficiency and reliability of this protocol as a function of network and transceiver parameters. To validate the accuracy of the analytical model, we compare the results from analysis with the simulation results.

As a second contribution of this paper, we propose a new dynamic probabilistic heuristic for MANETs. Although reliable broadcasts ensure the reachability of a broadcast to every node in the network, the reliable protocols have a very high communication overhead due to their dependency on 1-hop neighborhood information and sometimes on 2-hop neighbor information. Exchanging neighborhood information is a costly operation in MANETs and incurs a high overhead. The frequency of exchanging neighborhood information increases with the increase in mobility. Consequently, unreliable protocols are more preferable in highly mobile dynamic environment.

In highly mobile networks, a very simple but effective approach to reduce redundant rebroadcasts is the *probabilistic* broadcasting algorithm [5]. In probabilistic broadcasts, upon receiving a broadcast packet, each node in the network (except the sender and receiver) decides whether to rebroadcast it or not based on a pre-defined probability value. Mathematically speaking, instead of blindly forwarding any packet, a node in the network rebroadcasts a message with probability $P$ and takes no action with probability $1 - P$. The performance of such protocol highly depends on the selected value of the forwarding probability $P$. With higher $P$ values, more redundant transmissions take place but the chance of reaching all the nodes in the network is also increased. With lower $P$ values the opposite thing happens. Thus, an inherent problem of this approach is how to set a globally optimal probability value which is appropriate for all networking conditions and all dynamic environments. For example, in a *dense* network, a low probability value would ensure high reachability but the same low probability value would inhibit a significant number of nodes from receiving the broadcast in *sparse* networks. On the other hand, with a high probability value, high reachability can be maintained in sparse networks but the same value will create many redundant rebroadcasts in dense networks. So, the probability of each node should be assigned dynamically rather than statically based on the node density, distance from the sender and other network parameters.

While proposing the new probabilistic heuristic, at first we develop a mathematical equation for finding the rebroadcast probability of each node that depends on *additional* network coverage the node can provide. With higher additional coverage the rebroadcast probability becomes larger and with lower additional coverage the probability value also becomes smaller. The rebroadcasting probability of each node derived from the expression is then dynamically assigned to each node instead of a predefined *static* probability like the traditional probabilistic heuristics. We measure the efficiency and effectiveness of the proposed heuristic through extensive simulation experiments.

The rest of the paper is organized as follows. We present the background of our problem in Section II. Section III provides a brief discussion of prior related research works. Section IV develops the analytical model to characterize the self-pruning algorithm, while Section V describes the proposed dynamic probabilistic algorithm. Section VI presents simulation results to validate our model and measure effectiveness of the proposed new heuristic. Finally, Section VII concludes the paper with some pointers to possible future works.

## II. Background

### A. Self-pruning Broadcasting Algorithm

Self-pruning is a simple broadcasting algorithm which helps in reducing the redundant rebroadcast in flooding and minimizes the effect of broadcast storm problem. It uses neighborhood knowledge [9] for forwarding packets. Each node collects adjacent node information by periodically exchanging "hello" messages. In Fig. 1, node $u$ forwards a packet and $v$ receives the packet. Node $u$ piggybacks its neighbor list, $N(u)$ in the packet. Upon receiving the packet from $u$, node $v$ checks whether $N(v)$-$N(u)$-$\{u\}$ is empty. If it is empty, node $v$ is set as a non-forwarding node and refrains from forwarding. Otherwise, it is added to the forward list. These forward nodes, including the source node form a connected dominating set *(CDS)*. A set of nodes is a dominating set if every node in the network is either in the set or a neighbor of a node is in the set.

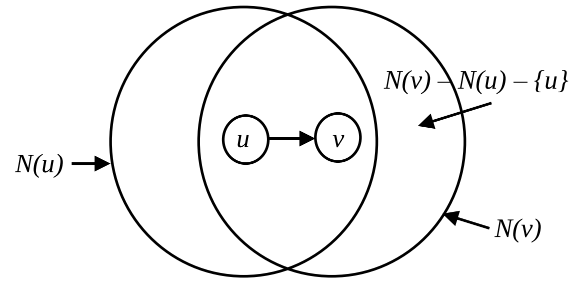


Fig. 1. Self-pruning method.

The additional area that can be benefited from node $v$'s transmission after receiving the packet from $u$ is denoted as the additional coverage area of $v$. In order to become eligible for forwarding, the node $v$ needs to have one or more nodes in its additional coverage area. If there is no node in the additional coverage area, then all neighbors of $v$ have already received the packet from $u$'s transmission. So, node $v$ does not need to rebroadcast. Consequently the node $v$ will be marked as a non-forwarding node.

### B. Probabilistic Broadcasting Algorithm

Probabilistic algorithm is a simple approach for broadcasting which also reduces redundant rebroadcasts. In the probabilistic scheme each node rebroadcasts with a predefined probability $P$. That means it assigns all nodes in the network the same probability $P$ for forwarding the message. When $P = 1$, it simply becomes blind flooding. The blind flooding is a straightforward approach that can guarantee reliable propagation over all regions of the network but it generates high number of redundant messages. When $P$ is set to a low value, the number of redundant message decreases but the scheme cannot guarantee message dissemination over the network. Probability $P$ must be high enough to propagate the message all over the network and also low enough to minimize the redundant messages in the network. Again, a static probability doesn't work same for both the sparse and dense network. For a sparse network, high probability is required to ensure high reachability. But in case of dense network, this high probability causes more redundant messages in the network. So, selecting a suitable probability $P$ for the whole network is critical without considering the node density, distance between nodes and other network parameters.

Notations used in this paper are listed in the Table I.

## III. Related Work

In this section, we will review some works related to our problem. A significant amount of works have been done to alleviate the broadcast storm problem. Lim and Kim [9] proposed a neighbor-knowledge based flooding approach dubbed

TABLE I. SUMMARY OF NOTATIONS.

| Notation | Description |
|---|---|
| $A$ | Network area |
| $n$ | Total node in the network |
| $\mu$ | Node density |
| $r$ | Transmission radius |
| *N(u)* | Set of neighbors of node *u* |
| *N(v)* | Set of neighbors of node *v* |
| $S_u$ | Circle areas covered by *u's* transmission range |
| $S_v$ | Circle areas covered by *v's* transmission range |
| $S_{v-u}$ | Additional cover region of *v* |
| $x$ | Distance between a node pair *(u, v)* |
| *C(u)* | Cover region of node *u* with radius *r* |
| $P_F(x)$ | Forwarding probability of node *v* at distance *x* from node *u* |
| $E_F$ | Expected number of neighbors forwarded |
| $F_F$ | Average fraction of neighbors forwarded |
| $P_{rand}$ | Some random number between 0 and 1 |

as *self pruning* which reduces the number of retransmission based on the existence of nodes in the additional coverage area. Wu and Dai [10] proposed a general framework for broadcasting based on self-pruning in ad hoc networks. In this approach, each node builds its k-hop information by exchanging periodical “Hello” messages. The “Hello” message also includes the priority of each node. Each node makes its forwarding decision based on two coverage conditions and these coverage conditions depend on the k-hop topology, priority of each node, and visited node information. The forward node set can be constructed by either a dynamic or static approach. If the protocol use visited node information then it is dynamic otherwise it is static. Wu and Dai [12] focused on the performance evaluation of existing self-pruning algorithms and also described an enhanced version of the generic protocol proposed by Wu and Dai [10]. In their work they have observed the efficiency and reliability of various protocols which have used different self-pruning conditions as a function of various network parameters. They have also evaluated the performance of the generic protocol under various network environments. But their analysis lacks theoretical details. Woon and Yeung [13] proposed two protocols, E-SBA and E-SBA+ which use a simple idea of delay timer to ensure that nodes with more uncovered neighbors rebroadcast earlier. These approaches solve the problems of an existing self-pruning protocol named SBA. They also introduced a timer suppression mechanism where a node upon receiving a rebroadcast message set a lower priority and reset the timer. Huang et al. [14] developed an analytical model for self-pruning. But they have evaluated their model based on only expected broadcast cost and scalability. The analysis of the effect of various network parameters, as well as, transceiver parameters was not considered in their work. It lacks proper analysis and evaluation of the mathematical model. Ni et al. [5] proposed several scheme, one of them is probabilistic rebroadcast to reduce the broadcast storm problem. In this approach the rebroadcast probability is set to a predefined value, *P* from 0 to 1. When a node receives a message for the first time it rebroadcast the message with probability *P* thus reducing the number of rebroadcast in the network. Kim et al. [15] developed a dynamic probabilistic broadcasting approach with coverage area and neighbor confirmation. Based on the coverage area of a node, probability is set. If it has small additional coverage area then a low probability is set and in case of large additional coverage, a high probability is set. The additional coverage is estimated by the distance from the sender. They have divided the coverage area of the sender into three sub area $A_1$, $A_2$ and $A_3$ with radii $r_1$, $r_2$ and $r_3$ respectively. A node can determine its coverage ratio depending on this three sub area. And by multiplying a sensitivity parameter, $\alpha$ with the coverage ratio rebroadcast probability is determined. They have also applied neighbor confirmation to prevent the early die out of a packet. But authors did not introduce any optimal value for the sensitivity parameter $\alpha$. Yassein et al. [16] presented a new probabilistic approach that dynamically adjusts the rebroadcast probability as per the node distribution and node movement. It used one-hop neighbor information to adjust probability of a node. If the message is received for the first time and the number of neighbors is less than the average number of neighbor then the node rebroadcast the message with a high probability value. Otherwise, if the node has a high degree neighbors its probability is set to low value. Bahadili [17] proposed a new probability adjusting model in which the number of first hop neighbors of the transmitting node is divided into three ranges (*low*, *medium* and *high*). And the rebroadcasting probability is adjusted according to three distribution functions ($f_{low}(k)$,$f_{med}(k)$ and $f_{hig}(k)$) where *k* is the number of first hop neighbors. Depending on the neighborhood density one of the three distribution function is selected and it returns one of the three rebroadcasting probability $p_{max}$, $p_{med}$ or $p_{min}$. The main drawback of this approach is that the estimation of the optimum values of the variables $p_{max}$, $p_{med}$ or $p_{min}$ and the appropriate value for the three ranges (*low*, *medium* and *high*) of neighbors is critical. They have also ignored the investigation of the effect of nodes density, nodes radio transmission range etc. on the performance of the new model.

The goal of all the above approaches is to minimize the number of rebroadcast in the network. To get a better perception of the simple self-pruning algorithm, we develop an analytical model for it. And also propose a dynamic probabilistic algorithm which uses a simple expression to calculate the rebroadcast probability.

## IV. ANALYTICAL MODEL FOR SELF-PRUNING

In this section, we mathematically analyze self-pruning algorithm and develop mathematical model for determining *redundancy*. At first we formally define redundancy. For our network model, we consider *n* nodes are uniformly distributed over a rectangular deployment area *A* and having homogeneous transmission range *r*. So, the average node density is: $\mu = n/A$.

**Redundancy.** The average fraction of 1-hop neighbors rebroadcasting/forwarding from a node's neighborhood is called redundancy. Mathematically:

$$\begin{aligned} &Redundancy \\ &= Average\,fraction\,of\,neighbors\,rebroadcasting \\ &= \frac{Average\,number\,of\,neighbors\,rebroadcasting}{Number\,of\,nodes\,in\,a\,node's\,TX\,area} \end{aligned} \quad (1)$$

As the node distribution is assumed to be uniform, it is easy to determine the number of nodes present in a node's

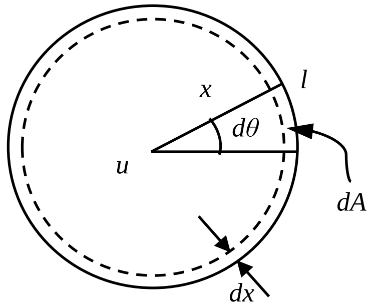


Fig. 2. A circle strip at distance *x*.

transmission (TX) area in Equation 1 if the node density is known a priori. To see how, let us observe an arbitrary node *u* within the deployment area. The average number of nodes located in the communication region of node *u* is:

$$\begin{aligned} N_R &= Node\,density \times Transmission\,Area \\ &= \mu \times \pi r^2 = \pi \mu r^2 \end{aligned} \quad (2)$$

Now let us find the average number of neighbors *rebroadcasting* in Equation 1. To find this quantity at first we need to derive $P_N(x)$, the probability that there exists a neighbor *v* at distance *x* from *u*. Clearly, $P_N(x) = 0$ if $x > r$. For $x \leq r$, we consider a small area strip defined by *dx* at the perimeter of the circle with radius *x* and centered at *u*. And, we also consider a small angle *dθ* which is measured from an arbitrary but fixed axis as shown in Fig. 2. The length of the arc $l = xd\theta$ and the area of the small region *dA* within this small strip can be approximated as, *dA* = *ldx* = *xdxdθ*. Therefore, the area of the entire small strip denoted by $A_{strip}$ becomes,

$$A_{strip} = \int_0^{2\pi} dA = \int_0^{2\pi} l dx = \int_0^{2\pi} x dx d\theta = 2\pi x dx \quad (3)$$

So, from Equation 3 $P_N(x)$ becomes,

$$\begin{aligned} P_N(x) &= Area\,of\,the\,strip \times Node\,density \\ &= A_{strip} \times \mu = 2\pi x dx \times \mu = 2\pi \mu x dx \end{aligned} \quad (4)$$

Once we find the probability of a node's existence at distance *x*, the next thing is to find the probability that the node will rebroadcast upon receiving a message from node *u* based on the rules of self-pruning. According to the self-pruning algorithm, a node *v* will rebroadcast if the node finds at least one new node in its *additional coverage* area. Let $P_F(x)$ be the probability that there exists at least one node in the additional coverage area *A.C.(x)* of node pair *u* and *v*.

The probability of any node *v* to be considered as a rebroadcasting node from *u*'s neighbor set, denoted by $P_E(x)$,is the probability that there exists a neighbor *v* at distance *x* from *u* multiplied by its rebroadcasting/forwarding probability $P_F(x)$. So $P_E(x)$ is:

$$P_E(x) = P_N(x) \times P_F(x) = 2\pi \mu x dx \times P_F(x) \quad (5)$$

The expected number of rebroadcasting neighbors of node *u* is found by integrating $P_E(x)$ from 0 to maximum transmission radius *r* within which node *u* possibly can communicate:

$$E_F = \int_0^r 2\pi \mu x dx \times P_F(x) dx \quad (6)$$

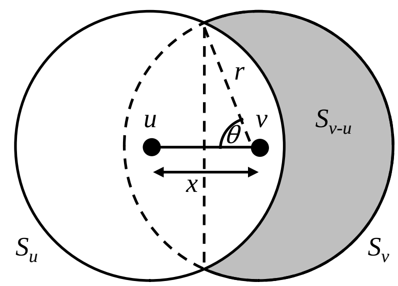


Fig. 3. Additional coverage area.

So, the average fraction of nodes rebroadcasting, denoted by $F_F$, can be found if we divide $E_F$ by $N_R$. Mathematically:

$$Redundancy = F_F = \frac{E_F}{\pi \mu r^2} = \frac{\int_0^r 2\pi \mu x \times P_F(x) dx}{\pi \mu r^2} \quad (7)$$

Now let us find the forwarding probability $P_F(x)$ in Equation 7. Recall that $P_F(x)$ is the probability that there exists at least one node in the additional coverage area *A.C.(x)* of node pair *u* and *v*. The additional coverage area is defined as follows.

Suppose an arbitrary node *u* sends a broadcast message and upon receiving the message node *v* decides whether to rebroadcast it or not. The distance between node *u* and node *v* is *x*. Let, $S_u$ and $S_v$ denote the circular areas covered by node *u's* and node *v's* transmission range, respectively. The *additional coverage* area that can be benefited from *v's* rebroadcast is the shaded region and denoted as S*v-u*. We consider a homogeneous system where every node in the network has the same transmission range. Let, *r* be the radii of $S_u$ and $S_v$, and *x* the distance between *u* and *v*. Fig. 3 shows the additional coverage area of node *v*, where *u* is the sending node and *v* is the receiving node. We can derive the additional coverage area,

$$|S_{v-u}| = |S_v| - |S_{u \cap v}| = \pi r^2 - INTC(x) \quad (8)$$

Here, ***INTC(x)*** is the intersection area of the two circles centered at two nodes separated by distance *x*.
Let us derive the equation for ***INTC(x)***,

$$\begin{aligned} INTC(x) &= 4 \int_{x/2}^{r} \sqrt{r^2 - x^2} dx \\ &= r^2 \times \left[ \pi - 2 \sin^{-1}\left(\frac{x}{2r}\right) - \sin 2 \sin^{-1}\left(\frac{x}{2r}\right) \right] \end{aligned} \quad (9)$$

So, using Equation 9, we can find the additional coverage area which is:

$$
\begin{aligned}
A.C.(x) &= \pi r^2 - INTC(x)\\
&= \pi r^2 - r^2 \times \left[\pi - 2\sin^{-1}\left(\frac{x}{2r}\right) - \sin 2\sin^{-1}\left(\frac{x}{2r}\right)\right]\\
&= \pi r^2 - \pi r^2 + r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right) + \sin 2\sin^{-1}\left(\frac{x}{2r}\right)\right]\\
&= r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right) + \sin 2\sin^{-1}\left(\frac{x}{2r}\right)\right]\\
&= r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right) + 2\sin\sin^{-1}\left(\frac{x}{2r}\right)\cos\sin^{-1}\left(\frac{x}{2r}\right)\right]\\
&= r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right) + \frac{x}{r}\sqrt{\cos^2\sin^{-1}\left(\frac{x}{2r}\right)}\right]\\
&= r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right) + \frac{x}{r}\sqrt{1-\sin^2\sin^{-1}\left(\frac{x}{2r}\right)}\right]\\
&= r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right) + \frac{x}{r}\sqrt{1-\left(\sin\sin^{-1}\left(\frac{x}{2r}\right)\right)^2}\right]\\
&= r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right) + \frac{x}{r}\sqrt{1-\frac{x^2}{4r^2}}\right]\\
&= r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right) + \frac{x}{2r^2}\sqrt{4r^2-x^2}\right] \qquad (10)
\end{aligned}
$$

The probability that a node exists in the additional coverage area *A.C.(x)* within the deployment area *A* is:

$$P_\Delta = \frac{A.C.(x)}{A} = \frac{A.C.(x)\times\mu}{n} \qquad (11)$$

The probability $P_k(A.C.(x))$ that exactly *k* nodes are located in the additional coverage area is:

$$P_k\left(A.C.(x)\right) = \binom{n-2}{k} P_\Delta^k \times (1-P_\Delta)^{n-2-k} \qquad (12)$$

Note that in Equation 12, *n-2* is used rather than *n*, because we exclude node *u* and node *v* from consideration. For large *n* and small$P_\Delta$, the binomial distribution can be approximated using Poisson distribution. So,

$$P_k\left(A.C.(x)\right) = \frac{(nP_\Delta)^k \times e^{-nP_\Delta}}{k!} \qquad (13)$$

The forwarding probability becomes the probability that there exists one or more nodes in the additional coverage area, *A.C.(x)*.

$$
\begin{aligned}
P_F(x) &= \sum_{k=1}^{n} P_k\left(A.C.(x)\right)\\
&= \sum_{k=1}^{\infty} \frac{(nP_\Delta)^k \times e^{-nP_\Delta}}{k!}\\
&= e^{-nP_\Delta}\left(\sum_{k=0}^{\infty}\frac{(nP_\Delta)^k}{k!} - \frac{(nP_\Delta)^0}{0!}\right)\\
&= e^{-nP_\Delta}\left(e^{nP_\Delta}-1\right)\\
&= 1 - e^{-nP_\Delta} \qquad (14)
\end{aligned}
$$

By plugging in the value of $P_F(x)$ into Equation 7, we get

$$
\begin{aligned}
Redundancy = F_F &= \frac{E_F}{\pi\mu r^2}\\
&= \frac{\int_0^r 2\pi\mu x \times \left(1-e^{-nP_\Delta}\right)dx}{\pi\mu r^2} \qquad (15)
\end{aligned}
$$

Finally using Equation 10, 11 and 15, we get

$$
\begin{aligned}
Redundancy = F_F &= \frac{\int_0^r 2\pi\mu x \times \left(1-e^{-n\frac{A.C.(x)\times\mu}{n}}\right)dx}{\pi\mu r^2}\\
&= \frac{\int_0^r 2\pi\mu x \times \left(1-e^{-A.C.(x)\times\mu}\right)dx}{\pi\mu r^2}\\
&= \frac{\int_0^r 2\pi\mu x \times \left(1-e^{-r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right)+\frac{x}{2r^2}\sqrt{4r^2-x^2}\right]\times\mu}\right)dx}{\pi\mu r^2}\\
&= \frac{2\int_0^r \left(x - xe^{-r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right)+\frac{x}{2r^2}\sqrt{4r^2-x^2}\right]\times\mu}\right)dx}{r^2} \qquad (16)
\end{aligned}
$$

According to Equation 16, the redundancy, $F_F$ is a function of network parameters namely, distance between two nodes *x*, transmission range *r*, and node density $\mu$. We use Simpson's 3/8 rule for the numerical integration of Equation 16. To achieve more accuracy of the mathematical result the step size *h* is set to a very small value (i.e., 0.001) for the numerical integration. From this equation, we can estimate average fraction of neighbors within a node's neighborhood forwarding the packet. The effect of various network parameters on the efficiency and reliability of the self-pruning algorithm can be easily determined by some simple analysis.

## V. Dynamic Probabilistic Algorithm

From Equation 14, we get the forwarding probability of a node *v* at distance *x* from node *u*, if there exists one or more nodes in the additional coverage area of *v*, *A.C.(x)*. Using Equation 10 and 11 into Equation 14, we get

$$
\begin{aligned}
P_F(x) &= 1 - e^{-nP_\Delta}\\
&= 1 - e^{-n\frac{A.C.(x)\times\mu}{n}}\\
&= 1 - e^{-A.C.(x)\times\mu}\\
&= 1 - e^{-r^2\left[2\sin^{-1}\left(\frac{x}{2r}\right)+\frac{x}{2r^2}\sqrt{4r^2-x^2}\right]\times\mu} \qquad (17)
\end{aligned}
$$

We use Equation 17 in dynamic probabilistic algorithm to find out the forwarding probability of a node. That means instead of a predefined probability for all the nodes in the network, every node will dynamically calculate its own rebroadcast probability based on this equation. Note that, the equation is a function of distance *x* between the sending node and the receiving node, node density $\mu$ and transmission range *r*. The proposed algorithm is described in Algorithm 1.

**Algorithm 1** Dynamic Probabilistic Broadcast

```
if Retrans(v, message) = 0 then        ▷ The node has not seen the message before.
    p_rand = rand()
    Calculate P_F(x)                    ▷ By using Equation 20.
    if p_rand ≤ P_F(x) then
        Retransmit message
    end if
end if
```

In Algorithm 1, *rand()* is a function which generates a random number between 0 and 1 and the value is stored in $p_{rand}$. And $P_F(x)$, rebroadcast probability of a certain node *v* located at distance *x* from node *u* is calculated using

Equation 17. If a node receives the message for the first time and the rebroadcast probability is greater than the generated random number then the node *v* rebroadcasts the message. Otherwise node *v* refrains itself from forwarding.

## VI. Simulation and Experiments

To validate the correctness of our analytical model, extensive simulation experiments have been conducted. We simulate the self-pruning algorithm and perform a comparative analysis based on the simulation results and the results derived from the analytical models. We also implement our proposed dynamic probabilistic algorithm and compare the performance of the algorithm with static probabilistic algorithm and an existing solution by Bahadili. We build the simulation program using C/C++.

### A. Simulation Scenario

We simulate a network where all nodes are randomly scattered in a $100m \times 100m$ square area. The density of node is varied by varying the number of nodes from 70 to 200 in the fixed deployment area. To analyze the effect of transmission range we vary the transmission range from $12m$ to $30m$. We only consider the connected networks for the simulation results. For the simulation experiments of self-pruning algorithm the results are averaged over 1000 random scenarios considering the source node can be anywhere in the network. And for the dynamic probabilistic algorithm we generate 100 random scenarios. For each of the scenarios, experiment is conducted considering each node as a source node and for each node in the network rebroadcast decision is calculated. Performance measures are reported as an average of these random samples.

### B. Performance Metrics

We analyze the performance of self-pruning using redundancy as a metric.

**Redundancy.** As mentioned earlier in Section IV, redundancy is defined as the average fraction of 1-hop neighbors rebroadcasting/forwarding from a node's neighborhood. Formally:

$$\begin{aligned} &Redundancy \\ &= Average\, fraction\, of\, neighbors\, rebroadcasting \\ &= \frac{Average\, number\, of\, neighbors\, rebroadcasting}{Number\, of\, nodes\, in\, a\, node's\, TX\, area} \end{aligned} \quad (18)$$

On the other hand, in order to evaluate the performance of the proposed dynamic probabilistic broadcast algorithm, we use two kinds of measures: redundancy and reachability. Redundancy is already defined. Reachability is defined below:

**Reachability.** The average fraction of the number of nodes receiving the broadcast packets to the total number of nodes in the network is defined as reachability. Mathematically:

$$\begin{aligned} &Redundancy \\ &= Average\, fraction\, of\, nodes\, receiving \\ &= \frac{Number\, of\, nodes\, receiving}{Number\, of\, nodes\, in\, the\, network\, area} \end{aligned} \quad (19)$$

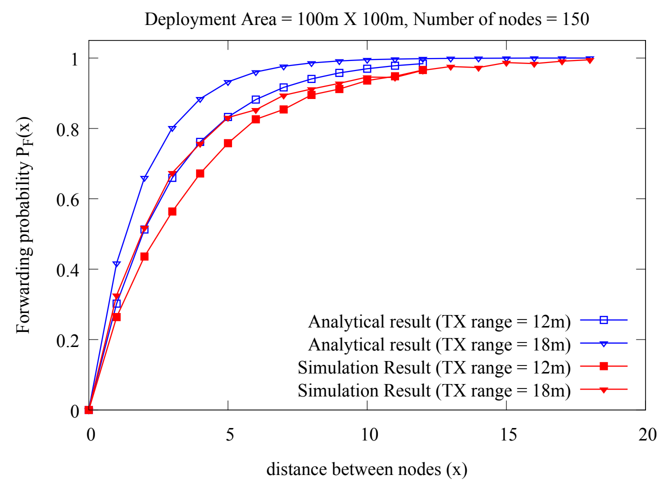


Fig. 4. Effect of distance between nodes, $x$ on $P_F(x)$.

### C. Results

In this section we present the simulation results and comment on various aspect of performance measures

*1) Analytical model of self-pruning:* To validate the correctness of our analytical model we compare the analytical result with our simulation result. Measurements from both simulation experiments and analytical expressions are plotted in the same graph.

*a) Effect of distance x on forwarding probability:* Effect of distance between two nodes, *x* on the forwarding probability is shown in Fig. 4. Distance between two nodes *x* has clear effects on the additional coverage region $S_{v-u}$, and thereby changes the forwarding probability. For a homogeneous transmission range *r*, additional coverage area increases with the increase of distance *x* and it reaches the highest value when *x* = *r*. Experiments are conducted for 150 nodes in the network and transmission range is set to *r* = $12m$ and *r* = $18m$. We vary the distance between nodes from 0 to maximum transmission range, *r* with an increment of $1m$ at each step. As can be seen from Fig. 4, the forwarding probability $P_F(x)$ exponentially increases with the increase in distance between two nodes and the forwarding probability gets the maximum value of 1 when *x* is equal to the transmission range *r*. Also for all scenarios, the results of analytical expressions are close to the simulation results. The discrepancy between the analytical results and simulation results are very small, maximal being around 8% when *r* = $12m$ and 12% when *r* = $18m$.

*b) Effect of Node Density on Redundancy:* We present the effect of node density on the average fraction of neighbors forwarding the packet with various transmission ranges while keeping the deployment area constant. The node density is varied by increasing 10 nodes in the network per step, starting from 70 to 200 and we examine the result for transmission ranges $15m$, $18m$ and $21m$. The node density, $\mu$ affects the number of nodes located in the additional coverage region $S_{v-u}$. With larger node densities, it is highly probable that at least one new node exists in the additional coverage area of *v* which makes it an eligible forwarding node. We present both the analytical results and results from simulation in

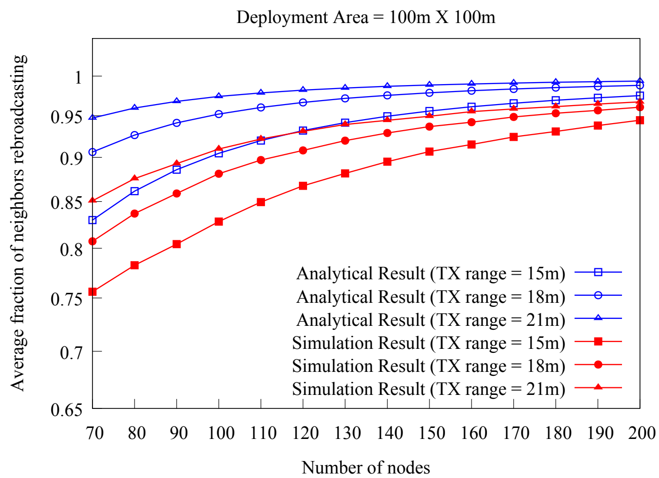


Fig. 5. Effect of node density on $F_F$.

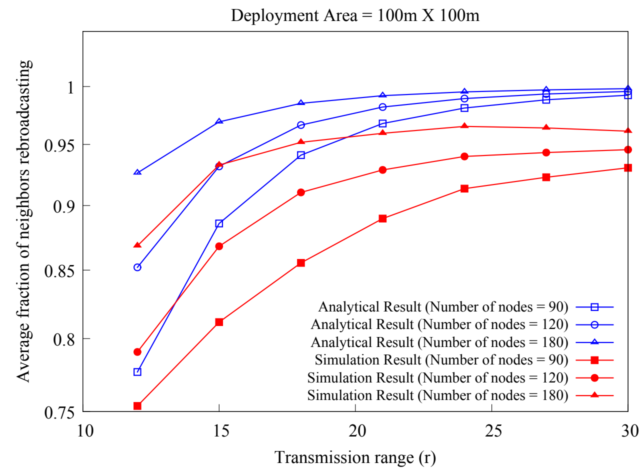


Fig. 6. Effect of transmission range on $F_F$.

Fig. 5. From the figure, we can see that average fraction of neighbors rebroadcasting, $F_F$ is increased with the increase in node density, $\mu$ in the network. That is, a higher fraction of neighbors is set as a forwarding node in more dense networks for all transmission ranges. The maximum difference between analytical results and simulation result is around 8%.

*c) Effect of Transmission Range on Redundancy:* To determine the effect of various transmission ranges on the average fraction of neighbors rebroadcasting, we measure $F_F$ with different node densities. Transmission range is varied between 12*m* to 30*m* with an increment of 3*m* at each step. We verify the result for $n$ = 90, $n$ = 120 and $n$ = 180 where, $n$ is the number of nodes in the network and the deployment area is constant. When the transmission range $r$ changes the circular regions $S_u$ and $S_v$ also change. Thus, the additional coverage region changes which results in changing of forwarding probability of a node. The result is presented in Fig. 6. From the figure, it is clear that average fraction of nodes rebroadcasting (i.e. $F_F$) increases with the increase of transmission range $r$. The analytical results and simulation results are very close. The small inaccuracy arises from the nodes located close to the perimeter of the deployment area. The nodes in the perimeter are more likely to have overlapping neighbors than the nodes in the center because their communication area is restricted.

*2) Proposed Dynamic Probabilistic Algorithm:* The simulation result of our proposed dynamic probabilistic algorithm is compared with the static probabilistic algorithm and plotted on the same graph for a clear comparison. We have also compared the performance of our proposed dynamic approach with the probability adjusting model proposed by Bahadili [17].

*a) Reachability:* We present the effect of node density on the reachability in Fig. 7. To analyze the performance of our proposed dynamic probabilistic algorithm we plot the result of both static and dynamic probabilistic broadcast algorithm in the same graph. The number of nodes in the network is varied from 70 to 200 and transmission range is set to 15*m*. Probabilities of values 0.5, 0.6, 0.7, 0.8 and 0.9 are considered for static probabilistic algorithm. As can be seen from the figure, in the dynamic probabilistic algorithm average fraction of nodes in the network receiving the packet is high, compared

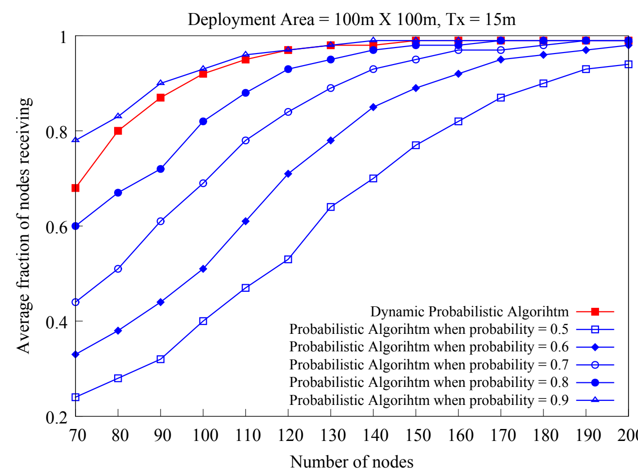


Fig. 7. Measurement of *Reachability* in Dynamic probabilistic algorithm and static probabilistic algorithm.

to the static probabilistic algorithm of values 0.5, 0.6, 0.7 and 0.8. The static values of 0.5, 0.6, 0.7 and 0.8 have a very low *reachability* in case of sparse networks. But our proposed scheme ensures high *reachability* for sparse, as well as, dense networks compared to the existing heuristic.

*b) Redundancy:* Finally, we present the effect of node density on the redundancy, i.e., on the average fraction of rebroadcast for both static and dynamic probabilistic algorithms. The result is shown in Fig. 8. Node density is varied by varying the number of nodes between 70 to 200 while keeping the deployment area constant at 100$m$ × 100$m$ square regions. The transmission range is set to 15*m*. For the static broadcast algorithm we vary the probability values from 0.5 to 0.9 with an increment of 0.1 at each step. As can be seen from the figure, the redundancy of the proposed dynamic broadcast is lower than the static probabilistic broadcast with probability value of 0.9. Although the redundancy of the dynamic broadcast is higher compared to the static broadcast with other probability values (i.e., $p$ = 0.5, 0.6, 0.7, 0.8), ultimately, the dynamic broadcast turns up the clear winner when we look at Fig. 7 and Fig. 8 together. To properly interpret the graphs in Fig. 8,

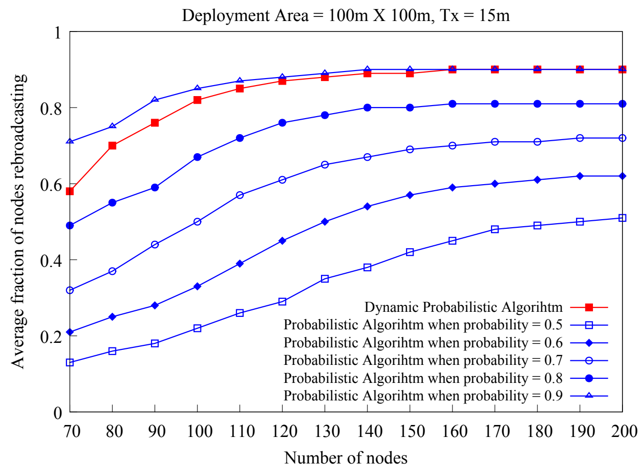


Fig. 8. Measurement of *Redundancy* in Dynamic probabilistic algorithm and static probabilistic algorithm.

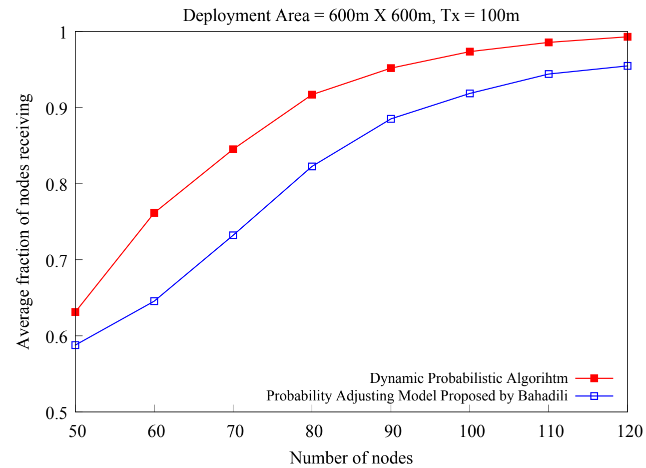


Fig. 9. Measurement of *Reachability* of the Proposed Dynamic probabilistic algorithm and probability adjusting model proposed by Bahadili.

we should realize that the spectacularly low redundancy of static broadcasting algorithm with probability values in the range of 0.5-0.8 results (at least partially) from the fact that the reachability of static broadcast in that probability range is also very poor (cf. Fig. 7). As the goal of every broadcasting algorithm is to maximize the reachability while minimizing the redundancy, the dynamic broadcasting algorithm incurs more redundancy simply to improve reachability.

*c) Comparison with an existing solution:* The comparison of the performance of our proposed dynamic approach with the probability adjusting model proposed by Bahadili [17] is shown in Fig. 9 and Fig. 10. For the experiment we have consider $600m \times 600m$ square area. The density of node is varied by varying the number of nodes from 50 to 120 in the fixed deployment area and the transmission range is set to $100m$. For the proposed scheme of Bahadili the number of first hop neighbors of the transmitting node, $k$ is divided into three ranges low (for $k \leq 4$), high (for $k \geq 15$) and medium ( for $4 < k < 15$). And the rebroadcasting probabilities $p_{max}$ and $p_{min}$ are set to 0.8 and 0.4. And , the value for $p_{med}$ is determined using Equation 20.

$$p_{med} = p_{max} - \frac{k - N_1}{N_2 - N_1}(p_{max} - p_{min}) \tag{20}$$

Fig. 9 presents the effect of node density on the *reachability* of our proposed dynamic approach with the probability adjusting model proposed by Bahadili. From the figure it is clear that our proposed scheme ensures high *reachability* for sparse, as well as, dense networks compared to the probability adjusting model by Bahadili.

Fig. 10 shows the effect of node density on the *redundancy*. As can be seen from the figure, the redundancy of the proposed dynamic broadcast is higher compared to the scheme of Bahadili. But we should realize that, in the proposed scheme of Bahadili [17] every node has to collect neighbor information by periodically exchanging "hello" messages because they need to know the number of 1-hop neighbors which will cause large message overhead in the network. On the contrary, in our proposed approach every node can calculate its rebroadcast

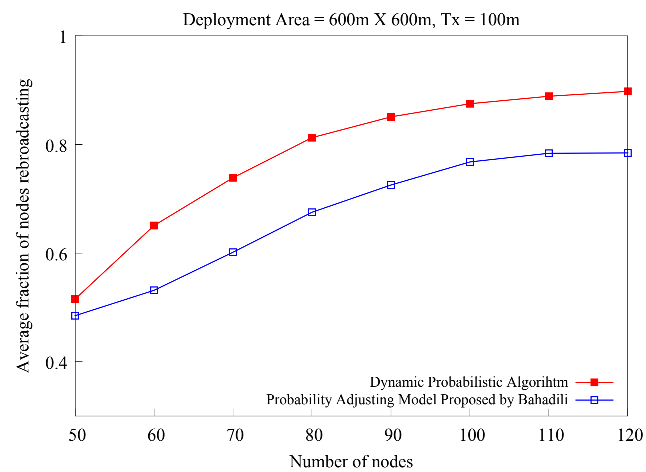


Fig. 10. Measurement of *Redundancy* of the Proposed Dynamic probabilistic algorithm and probability adjusting model proposed by Bahadili.

probability without the aid of any neighbor information. And the higher reachability in our proposed approach is another cause for the higher redundancy. So, we can say that our proposed approach performs better than the approach proposed by Bahadili [17].

## VII. Conclusion and Future Work

In this paper, we have presented an analytical model for characterizing self-pruning algorithm as a function of various network and transceiver parameters and get a better perception of the complex mechanism of the algorithm. With the help of this model, we can easily estimate the reliability and performance of the network prior to the network deployment. Moreover, the analysis helps us to design new heuristics for broadcasting in wireless ad hoc networks. In this paper, we have also introduced a dynamic probabilistic algorithm based on the rebroadcasting probability of a node. The rebroadcast probability is calculated using the expression of forwarding probability of a node derived during the modeling of self-pruning algorithm. Simulation results show that, the proposed algorithm performs better than the static probabilistic algorithm and the probability adjusting model proposed by Bahadili. In future we will design a more adjustable dynamic probabilistic algorithm which will be able to achieve more saved rebroadcast and higher reachability in the network.

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