Impact of Polarization Mode Dispersion on CPFSK Transmission Systems Using MZI Based Direct Detection Receiver

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Abstract—An analytical approach is presented to determine the impact of signal phase distortion due to polarization mode dispersion (PMD) in a single mode fiber (SMF) on the bit error rate (BER) performance of an optical continuous phase FSK (CPFSK) transmission system with MZI based direct detection receiver. The probability density function (pdf) of the random phase fluctuation due to PMD and group velocity dispersion (GVD) at the output of the receiver is determined analytically. Based on the pdf of the random phase fluctuation the BER performance results are evaluated at a bit rate of 10 Gb/s fiber with dispersion co-efficient $D_c = 15$ ps/km-nm for different values of the mean differential group delay (DGD). The computed results show that the direct detection CPFSK system suffers a significant amount of power penalty due the effect of PMD at a bit rate of 10 Gb/s. At a modulation index of 0.5, the penalty at a BER of $10^{-9}$ is found to be 0.5 dB, 1.10 dB, 2.10 dB and 3.50 dB when the mean differential group delay (DGD) is 20 ps, 30 ps, 40 ps and 50 ps respectively at modulation index of 0.5 for a fiber length of 100 km. At modulation index of 1.0, the penalty is approximately 4.10 dB for a mean DGD of 40 ps. The penalty is more significant at higher modulation index and bit rate.

Index Terms— Bit-error rate, polarization mode dispersion, probability density function, differential group delay.

I. INTRODUCTION

Optical continuous-phase FSK (CPFSK) is an attractive modulation format as it allows generation of compact spectra that allows for receiver envelope detection by properly selecting the modulation index [1]. However, the effect of PMD constitutes one of the main limiting factors for reliable optical fiber system performance at gigabits transmission rates. As the two polarization fields move at different group velocities with a differential group delay (DGD), there is a fluctuation in signal phase between the two principal states of polarization which causes a random phase fluctuation of the signal itself resulting in spectral broadening of the transmitted signal that leads to bit pattern corruption and higher bandwidth requirement, and eventually contributes to BER deterioration, performance variation or system fading even at moderate bit rate [2]-[5].

The effect of PMD has been the subject of considerable research interests during the past few years and the developed ideas are briefly described in a series of publication [6]-[12]. The effect for PMD on the error probability is theoretically assessed at a bit rate of 2.5 Gb/s for an IM-DD system and the BER in a IM/DD has been shown to strongly depend on the state of polarization and DGD [6]. Some studies treated PMD as deterministic term in numerical solution or are based on simple analytical approach of determining the conditional bit error probability [7]. The simulation results on the effects of PMD on an amplified IM-DD system is reported in Ref. [8] as a function of the DGD. The effect of PMD on the bit error rate performance of heterodyne FSK system is also reported [13]. Recently, the performance of optical DPSK system with direct detection receiver is reported in presence of PMD [14]. Several attempts have been reported to compensate the effect of PMD in electronic as well as in optical domain [15]-[17]. Although there are many approaches reported to minimize the effect of PMD, there is no analytical development to evaluate the impact of PMD on the BER performance of an optical transmission system.

In this paper, we provide an analytical approach to evaluate the BER performance limitations of an optical direct detection CPFSK system impaired by PMD. The method is based on the linear approximation of the output phase of a linearly filtered angle-modulated signal such as the CPFSK signal. The expression for the overall degradation in signal phase is derived following an analytical approach by using an equivalent transfer function of a single fiber which includes the effects of PMD in presence of group velocity dispersion (GVD). The conditional BER is then determined using the pdf’s of the random phase fluctuation due to PMD and GVD at the receiver output in the presence of receiver noise for...
different values of mean DGD. Power penalty suffered by the system due to PMD is evaluated at a bit rate of 10 Gb/s.

II. SYSTEM MODEL

The block diagram of an optical CPFSK transmission system with direct detection model is shown in Fig.1. Fig.2 shows the low pass equivalent CPFSK system model considering PMD and chromatic dispersion. The signal components in the two output PSPs propagate independently through the entire system from the modulator to the balanced receiver. In the transmitter, non-return to zero (NRZ) data at 10 Gb/s is used to directly modulate a laser to generate the CPFSK signal that is transmitted through a single mode fiber. At the receiving end, MZI based direct detection receiver detects the received optical signal. The fields corresponding to the two PSPs are separately detected and the resulting photocurrents are summed. We decompose the entire system into a pair of subsystems, each corresponding to one PSP. We sum the outputs of the two subsystems to obtain sampled decision variables \( i_m(t) = i_{m1}(t) + i_{m2}(t) \). The output photocurrent is filtered by a low pass filter and fed to a sampler followed by a comparator. The threshold voltage of the comparator is set to zero value. If the output voltage is greater than zero a binary ‘1’ is detected, otherwise a binary ‘0’ is detected.

![Fig. 1. Block diagram of an optical CPFSK transmission system with MZI based direct detection receiver](image1)

![Fig. 2. Low pass equivalent direct detection CPFSK system model considering PMD and chromatic dispersion](image2)
III. Theoretical Analysis

The complex electric field at the output of the CPFSK transmitter and input to the fiber is represented as:

\[ E_1(t) = \sqrt{2P_T} \exp[j(2\pi f_c t + \phi_s(t))][c_1.e_1] \]  

\[ E_2(t) = \sqrt{2P_T} \exp[j(2\pi f_c t + \phi_s(t))][c_2.e_2] \]  

where \( f_c \) is the carrier frequency, \( P_T \) is the transmitted optical power and the CPFSK modulating phase \( \phi_s(t) \) is given by

\[ \phi_s(t) = 2\pi f_d \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} a_k p(t - kT)dt + \phi_n(t) \]  

where \( a_k \) is the random NRZ bit, \( f_d \) is the peak frequency deviation, \( p(t) \) is the elementary pulse shape and \( \phi_n(t) \) is the phase noise of the transmitting laser. Here \( c_1, c_2 \) represent unit vectors and \( e_1, e_2 \) represent the two principal states of polarization respectively. The electric field at the output of the fiber is then given by:

\[ E_{01}(t) = \sqrt{2P_T} \exp[j(2\pi f_c t + \phi_s(t))][c_1.e_1] \otimes h_1(t) \]  

\[ E_{02}(t) = \sqrt{2P_T} \exp[j(2\pi f_c t + \phi_s(t))][c_2.e_2] \otimes h_2(t) \]  

where \( h_1(t) \) and \( h_2(t) \) are the inverse Fourier transform of the fiber transfer function \( H_1(f) \) and \( H_2(f) \) respectively which include the effect of polarization mode dispersion and group velocity dispersion (GVD) and \( \otimes \) denotes convolution. The low pass equivalent expression for \( H_1(f) \) and \( H_2(f) \) are given by [14],

\[ H_1(f) = \sqrt{\alpha} \exp[j2\pi f(-\Delta r/2) - j\gamma(f\pi T)^2] \]  

\[ H_2(f) = \sqrt{1-\alpha} \exp[j2\pi f(\Delta r/2) - j\gamma(f\pi T)^2] \]  

where \( \Delta r \) represents the differential group delay (DGD) between the two principal states of polarization (PSP), \( \alpha \) is the PMD power-splitting ration and \( \gamma \) is the chromatic dispersion index. Here we assume that there is negligible polarization-dependent loss and we use the principal states model to characterize first-order PMD. Assuming linear phase approximation the output electric fields for the two polarization states can be written as

\[ E_{01}(t) = \sqrt{2P_s} \exp[j(2\pi f_c t + \phi_{s1}(t) + \theta_{n1}(t))][c_1.e_1] \]  

\[ E_{02}(t) = \sqrt{2P_s} \exp[j(2\pi f_c t + \phi_{s2}(t) + \theta_{n2}(t))][c_2.e_2] \]  

where,

\[ \phi_{s1}(t) = \text{Re}[\phi_s(t) \otimes h_1(t)] \]  

\[ \phi_{s2}(t) = \text{Re}[\phi_s(t) \otimes h_2(t)] \]  

\[ = 2\pi f_d \int_{-\infty}^{\infty} \sum_{k} a_k \text{Re}[p(t - kT) \otimes h_1(t)]dt \]  

\[ \phi_{s2}(t) = \text{Re}[\phi_s(t) \otimes h_2(t)] \]  

\[ = 2\pi f_d \int_{-\infty}^{\infty} \sum_{k} a_k \text{Re}[p(t - kT) \otimes h_2(t)]dt \]  

In the FSK direct detection receiver with Mach-Zehnder interferometer (MZI), the MZI act as an optical discriminator and differentially detect the ’mark’ and ’space’ of the received FSK signal, which are then directly fed to a pair of photo-detectors. The transmittances of the two branches of the MZI are \( T_1(t) \) and \( T_2(t) \), and \( \tau \) is the time delay between the two branches. For a ‘mark’ transmission, the current at the output of the photo-detectors can be expressed as:

\[ i_{m1}(t) = R_d P_s \cos[2\pi f_c t + \phi_{s1}(t) + \Delta\theta_{n1}(t,\tau)][c_1.e_1] \]  

\[ i_{m2}(t) = R_d P_s \cos[2\pi f_c t + \phi_{s2}(t) + \Delta\theta_{n2}(t,\tau)][c_2.e_2] \]  

where \( \Delta\theta_{n1}(t,\tau) = \phi_{s1}(t) - \phi_{s1}(t - \tau) \) \( \Delta\theta_{n2}(t,\tau) = \phi_{s2}(t) - \phi_{s2}(t - \tau) \)

For a ’mark’ transmission and ideal CPFSK demodulation condition (assuming NRZ data), we have \( 2\pi f_c \tau = (2n + 1)\frac{\pi}{2} \), where \( n \) is an integer; and

\[ 2\pi f_d \tau = \frac{\pi}{2} \]  

Thus, the currents at the output of photodetector can be expressed as,

\[ i_{m1}(t) = R_d P_s \cos[2\pi f_c t + \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} q_1(t) + \]  

\[ \frac{\pi}{2} \sum_{k \neq 0} a_k q_1(t - kT) + \Delta\theta_{n1}(t,\tau)][c_1.e_1] \]  

\[ i_{m2}(t) = R_d P_s \cos[2\pi f_c t + \frac{\pi}{2} - \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} q_2(t) + \]  

\[ \frac{\pi}{2} \sum_{k \neq 0} a_k q_2(t - kT) + \Delta\theta_{n2}(t,\tau)][c_2.e_2] \]  

where,

\[ q_1(t) = \frac{1}{\tau} \int_{t-\tau}^{t} g_1(t)dt \]  

\[ g_1(t) = \text{Re}[p(t) \otimes h_1(t) \otimes h_{LP}(t)] \]  

\[ q_2(t) = \frac{1}{\tau} \int_{t-\tau}^{t} g_2(t)dt \]  

\[ g_2(t) = \text{Re}[p(t) \otimes h_2(t) \otimes h_{LP}(t)] \]  

where \( h_{LP}(t) \) represents the impulse response of the electrical low pass filter, which is assumed to be fifth-order Bessel function type. The transfer function of the low pass filter is given by,

\[ H_{LP}(f) = \frac{945}{jf^3 + 15f^4 - 105jf^2 - 420f^2 + 945jf + 945} \]
where $F = 2.43f / B_e$ and $B_e$ is the 3-dB cutoff frequency.

The output phase distortion due to PMD and GVD is given by,

$$\Delta \phi_1 (t, \tau, \Delta \tau) = -\frac{\pi}{2} + \frac{\pi}{2} q_1(t) + \frac{\pi}{2} \sum_{k \neq 0} a_k q_1(t-kT)$$

$$\Delta \phi_2 (t, \tau, \Delta \tau) = -\frac{\pi}{2} + \frac{\pi}{2} q_2(t) + \frac{\pi}{2} \sum_{k \neq 0} a_k q_2(t-kT)$$

$$= \Delta \phi_{x1}(t, \tau) + \xi_1(t, \tau)$$  \hspace{1cm} (19)

$$= \Delta \phi_{x2}(t, \tau) + \xi_2(t, \tau)$$  \hspace{1cm} (20)

where $\Delta \phi_{x1}(t, \tau)$ and $\Delta \phi_{x2}(t, \tau)$ are mean output phase error; $\xi_1(t, \tau)$ and $\xi_2(t, \tau)$ are the random output phase fluctuation due to ISI bit pattern caused by PMD and GVD.

The total signal current at the output of LPF corresponding to a ‘mark’ is given by,

$$i_m(t) = R_d P_s x_1(t) + R_q P_q x_2(t)$$  \hspace{1cm} (21)

where, $x_1(t)$ and $x_2(t)$ describe the phase noise induced by the interferometer intensity due to random phase fluctuation and can be expressed as,

$$x_1(t) = \cos[\Delta \phi_1(t)]$$  \hspace{1cm} (22)

$$x_2(t) = \cos[\Delta \phi_2(t)]$$  \hspace{1cm} (23)

The low-pass filter output for ‘mark’ transmission, at a sampling time $t$ is,

$$i_m(t) = 2R_d P_s \left[ \cos(\Delta \phi) \cos(\Delta \phi) \right] + n(t)$$  \hspace{1cm} (24)

where $\Delta \phi = \Delta \phi_1(t, \tau, \Delta \tau) + \Delta \phi_2(t, \tau, \Delta \tau)$

and $\Delta \phi = \Delta \phi_1(t, \tau, \Delta \tau) + \Delta \phi_2(t, \tau, \Delta \tau)$

If we consider the mean phase distortion only then the conditional bit error rate can be expressed as,

$$P(e | \Delta \phi, \Delta \phi) = \frac{1}{2} \text{erfc} \left( \frac{Q}{\sqrt{2}} \right)$$  \hspace{1cm} (25)

where, $Q = \frac{2R_d P_s \left[ \cos(\Delta \phi) \cos(\Delta \phi) \right]}{\sigma_m}$  \hspace{1cm} (26)

The contribution of the term (random phase fluctuation) $\xi_1(t, \tau)$ of (19) and $\xi_2(t, \tau)$ of (20) can be evaluated by finding the probability density function (pdf). Assuming $\{ a_k \}$ are independent and identically distributed (IID) random variables, the characteristic function of $\xi_1$ and $\xi_2$ is given by,

$$\Phi_{\xi_1}(j\xi_1) = \prod_{i=1}^{\infty} \cos[q_1(t)] = \sum_{i=1}^{\infty} \left( \frac{2i}{2} \right)^{-1} M_{2i}$$  \hspace{1cm} (27)

$$\Phi_{\xi_2}(j\xi_2) = \prod_{i=1}^{\infty} \cos[q_2(t)] = \sum_{i=1}^{\infty} \left( \frac{2i}{2} \right)^{-1} M_{2i}$$  \hspace{1cm} (28)

where, $q_1(t) = |q_1(t-iT)|$, $q_2(t) = |q_2(t-iT)|$, $M_{2i}$ and $M_{2i}^*$ are even order moments of the characteristic function of random variable $\xi_1$ and $\xi_2$ respectively. By using the above two equations the pdf $P_{\Delta \phi}(\delta \phi)$ of $\delta \phi$ and the pdf $P_{\Delta \phi}(\Delta \phi)$ of $\Delta \phi$ can be evaluated following Ref. [13].

For a given value of mean differential group delay (DGD) $\Delta \tau$, the conditional BER can be expressed as:

$$BER(\Delta \tau) = \int_{-\infty}^{\infty} P(e | \Delta \phi, \Delta \phi) P_{\Delta \phi}(\delta \phi) \, d(\delta \phi)$$  \hspace{1cm} (29)

The average error probability can be evaluated as

$$BER = \int_{-\infty}^{\infty} P_{\Delta \phi}(\Delta \tau) d(\Delta \tau)$$  \hspace{1cm} (30)

where, $P_{\Delta \phi}(\Delta \tau)$ is the pdf of $\Delta \tau$ which has Maxwellian distribution with $\Delta \tau_m$ as quadratic mean of $\Delta \tau$ [8].

IV. RESULTS AND DISCUSSION

Following the analytical approach, the bit error rate performance results for direct detection CPFSK receiver with post detection low pass filter are evaluated at a bit rate of 10 Gb/s with several values of mean DGD and fiber lengths.

Fig. 3. Conditional BER versus received power, $P_0$ for direct detection CPFSK system impaired by PMD (modulation index, $h=0.50$)
The plots of conditional BER versus received optical power $P_s$ are shown in Fig. 3 for mean DGD of 10 ps, 20 ps, 30 ps, 40 ps, 50 ps and 60 ps with fiber length of 100 km and modulation index of 0.50 and plots for modulation index 1.0 are shown in Fig. 4.

The results show that the BER is highly degraded when the mean DGD is higher for a given fiber length and the system suffers a significant amount power penalty at BER $= 10^{-9}$ which is found to increase with increasing value of the mean DGD.

![Figure 3](image1.png)

The amount of power penalty is found to be approximately 0.50 dB, 1.10 dB, 2.10 dB and 3.50 dB for mean DGD of 20 ps, 30 ps, 40 ps and 50 ps respectively when the modulation index is 0.50. It is further observed that the penalty is much higher at higher modulation index such as $h=1$ compared to $h=0.50$ i.e., the penalty is approximately 4.10 dB when mean DGD is 40 ps and $h=1.0$.

It is further noticed that penalty suffered by direct detection CPFSK is lower than that of NRZ-OOK system. But at higher modulation index penalty suffered by CPFSK with $h=0.50$ is higher than that of OOK and DPSK system. For example, the penalty is 1.35 dB and 2.40 dB corresponding to DGD/bit duration ratio ($\Delta\tau/T$) are 0.20 and 0.30 respectively for direct detection CPFSK system at modulation index 1.0. The corresponding penalty values are 0.50 dB, 1.35 dB and 0.30 dB, 0.80 dB for NRZ-OOK and NRZ-DPSK respectively.

![Figure 4](image2.png)

Fig. 4. Conditional BER versus received power, $P_s$ for direct detection CPFSK system impaired by PMD (modulation index, $h=1.0$)

![Figure 5](image3.png)

Fig. 5. PMD-induced power penalty as a function of DGD/bit duration ratio ($<\Delta\tau>/T$) for NRZ- and RZ-OOK, NRZ-2DPSK and NRZ-CPFSK system.

V. CONCLUSION

An analytical approach is presented to evaluate the impact of PMD on the bit error rate performance of direct detection CPFSK system. The results show that the penalty due to PMD suffered by the CPFSK system is significant at higher modulation index and higher value of the DGD between two-polarization modes. A direct-detection CPFSK system with modulation index of 0.50 suffers almost same amount of power penalty as direct detection NRZ-OOK system.

VI. REFERENCES


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