Chordal Graphs

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Outline

1. Chordal Graphs: Definition
2. Chordal Graphs: Recognition
3. Chordal Graphs: Clique Tree Representation
Chordal Graphs

A chord in a cycle is an edge which goes between two vertices which are not consecutive in the cycle.

A graph $G$ is chordal if there are no chordless cycles in $G$ of length greater than three.

Chordal graph always contain a vertex $v$ such that the neighborhood of $v$ is a clique. Such a vertex is called a simplicial vertex.

A perfect elimination scheme $v_1, v_2, v_3, \ldots, v_n$ is an ordering of the vertex set if and only if for all $i$, $v_i$ is simplicial in the graph induced by $v_{i+1}$ through $v_n$. 
The number of chordal graph is $2^{\Omega(n^2)}$.

**Proof.** Chapter 15
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Recognition of Chordal Graphs

Trivial algorithm $O(n^4)$ time.
Recognition of Chordal Graphs

Trivial algorithm $O(n^4)$ time. $G$ is chordal if and only if there is a perfect elimination scheme for $G$.

Linear Algorithm:

- Construction phase: a construction phase which creates an ordering which is a perfect elimination scheme if and only if $G$ is chordal.
- Verification phase: a verification phase which checks whether the ordering which was constructed is in fact a perfect elimination scheme.
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1. Chordal Graphs: Definition
2. Chordal Graphs: Recognition
3. Chordal Graphs: Clique Tree Representation
Chordal graphs correspond exactly to intersection graphs of subtrees of a tree. This model is called clique tree model. A tree $T$ is a clique tree of a graph $G$ if the nodes of $T$ correspond to maximal cliques of $G$ and each vertex $v$ of $G$ corresponds to a subtree of cliques which contain $v$. 
Construction Clique Tree Representation

Assume that vertices are labeled from 1 to $n$ according to their position in a perfect elimination scheme. We construct the clique tree for the graph induced on vertices $i$ through $n$ for all vertices, starting with $i = n$ and ending with $i = 1$.

Let $C(v)$ be the clique consisting of $v$ and all neighbors of $v$ which appear after $v$ in the elimination scheme. After each vertex $v$ is processed, $v$ is given a pointer to $C(v)$. Note that vertices may be added to this clique later in the algorithm, but $v$ will always point to a clique which contains $C(v)$.

Let $i$ be the next vertex considered, and assume we know the clique tree on the graph induced by $i + 1, \cdots, n$. We need to add $C(i)$ to the clique tree.
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Let $j$ be the first vertex of $C(i)$ in the elimination ordering, other than $i$ itself. If $|C(i)| = 1 + |C(j)|$, and the clique pointed to by $j$ is equal to $C(j)$, we add $i$ to this clique. Otherwise, add $C(i)$ as a new node of the tree. Connect $C(i)$ to the tree by adding an edge from $C(i)$ to the clique pointed to by $j$. 